Product-Line Design in the Presence of Consumers’ Anticipated Regret

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Abstract

Consumers are often uncertain about their valuations for product quality when choosing among different options in a product line, and will learn their valuations only after making a purchase. Some consumers may thus experience over-purchase or under-purchase regret, depending on whether they have purchased a higher or lower quality level than the level that they would have chosen had they known their true valuations. When consumers anticipate their potential post-purchase regret, their purchase decisions may be affected. Our analysis shows that over-purchase regret lowers the firm’s profit but under-purchase regret can benefit the firm. When the firm optimally designs its product line, the quality difference between its offerings will be larger (smaller) if consumers’ anticipated regret increases (reduces) its profit. Surprisingly, consumers may be better off with anticipated regret than when they are regret-free. We also examine how the firm can improve its profit by alleviating consumers’ anticipated regret through offering free trials before purchase or allowing consumers to swap their products on an exchange platform post purchase.

Key words: anticipated regret, product line design, quality, behavioral economics, price discrimination

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1. Introduction

Consumers sometimes regret their purchase decisions when they learn a mismatch of the product with their preferences after using the product. One important reason is that consumers may be uncertain about their true need or valuation for product quality at the time of purchase, and this uncertainty can be resolved only after purchasing and fully experiencing the product. Indeed, extensive research in psychology and economics has shown that the magnitude of consumer regret can be significant (e.g., Simonson 1992, Zeelenberg 1999). Furthermore, consumers may anticipate their future regret and will make tradeoffs to minimize or mitigate their anticipated regret when making purchase decisions (e.g., Kahneman and Tversky 1982a, 1982b, Landman 1987).

Realizing the importance of consumers’ potential post-purchase regret, some firms are actively trying to address this concern. For example, Chrysler adopted a “Regret Free Pledge” campaign that allows free returns within 60 days of the new purchase.² Fred Diaz, president and chief executive officer of Ram Truck Brand and lead executive for U.S. sales, claimed that “with this pledge, consumers will have the confidence to know they made the right purchase or they can return the vehicle, no questions asked.” Similarly, in the advertisement “No asterisk, no regret,” T-Mobile also reminds consumers of their potential frustration of running out of wireless data. The key message of the ad is that consumers can get rid of their regret by choosing the iWireless plan with a large allowance of high-speed data.³

There are two distinct types of regret in the context of purchasing vertically differentiated products, depending on whether consumers have purchased a lower or a higher quality level than the ideal quality level that they would have chosen in the absence of their uncertainty. We denote the first type of regret “under-purchase regret,” where a consumer purchased the low-quality product but ex post found out that she could have gained a higher utility if she had purchased the high-quality version. By contrast, we denote the second type of regret “over-purchase regret,” where a consumer purchased the high-quality product but ex post learned that she could have been better off if she had purchased the low-quality version. In this paper, we explicitly model the two types of regret and analyze their impact on consumers’ behavior and the firm’s response. It turns

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² Refer to http://www.autoremarketing.com/subprime/consumers-can-now-select-special-chrysler-incentives
out that these two different types of anticipated regret have differential impacts on a firm’s optimal product-line strategies.

To demonstrate the distinction between the two types of regret and to measure their intensities, we conducted an online survey of 107 participants about their possible regret when they under-utilized a product bought earlier (the case of over-purchase regret) or when they needed a higher quality version than what they have purchased (the case of under-purchase regret). This survey shows that consumers display a moderate to significant amount of post-purchase regret across three product categories, including fitness trackers, TVs and smart phones. The average extent of post-purchase regret ranges from 2.47 to 3.36 on a scale between 1 and 5, with 1 being minimal regret and 5 being maximal regret. On the one hand, 79%/94%/79% of consumers indicated a certain level of over-purchase regret (with ratings of 2 or above) in the case of fitness trackers, TVs, and smart phones, respectively. On the other hand, 94%/90%/98% of them displayed a certain level of under-purchase regret in these product categories. Furthermore, in the case of a fitness tracker and a smart phone, consumers reported a stronger level of under-purchase regret than over-purchase regret ($M_{\text{fitness tracker}}^{\text{over}} = 2.47, M_{\text{fitness tracker}}^{\text{under}} = 3.35, t = 4.16, p < 0.001$; $M_{\text{smartphone}}^{\text{over}} = 2.68, M_{\text{smartphone}}^{\text{under}} = 3.28, t = 2.80, p = 0.006$). When it comes to the decision to TV purchase, consumers displayed a stronger over-purchase regret than under-purchase regret ($M^{\text{over}} = 3.36, M^{\text{under}} = 2.93, t = 2.11, p = 0.037$).

Given the ubiquity of post-purchase regret in consumers’ decision making, we intend to address the following research questions: 1) How does consumers’ anticipated regret influence a firm’s optimal pricing and profits in its product line? 2) How does consumers’ anticipated regret affect consumer surplus? 3) When a firm can strategically choose product quality in the product line, how will consumers’ anticipated regret affect the firm’s optimal quality decision? Towards this end, we build an analytical framework to study the effect of consumers’ anticipated regret on a firm’s product-line decisions. In our model, a monopoly firm sells a product line consisting of a high-quality product and a low-quality one. Consumers are uncertain about their need or valuation

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4 See Figure A1 in the appendix.
5 Statistical significance ($p < 0.05$) remains after we controlled for respondents’ age, gender, and self-reported technology-savviness.
for product quality prior to purchase, and can resolve the uncertainty only after purchasing and using the product. Consumers may experience a disutility associated with both under-purchase regret and over-purchase regret, and they anticipate their potential regret when making purchase decisions. Consumers are heterogeneous in their likelihood of realizing a high valuation of product quality after purchase, and both the firm and consumers ex ante know the distributions of different types of consumers.

We highlight a few main findings from our analysis. First, the firm may obtain a higher profit if consumers anticipate potential post-purchase regret when making purchase decisions. In fact, although consumers’ level of over-purchase regret always decreases the firm’s profit, the firm can benefit from consumers’ under-purchase regret. The latter happens because a stronger under-purchase regret gives buyers of the high-quality product less incentive to consider the low-quality product, because the low-quality product is more likely to result in a greater disutility from under-purchase regret.

Second, our analysis shows that consumer surplus decreases in either type of regret when product quality is exogenous, but it may increase when the firm endogenously chooses its quality levels. Furthermore, when consumers’ under-purchase regret is sufficiently strong, the firm can extract the entire surplus from both the high-quality and the low-quality product buyers. This contrasts the classic result that a firm has to leave positive surplus to the high-valuation consumers in the absence of anticipated regret.

Third, when the firm can endogenously choose the quality of its products, it will increase the quality gap in its product line when consumers have stronger under-purchase regret, if and only if the increased under-purchase regret can increase the firm’s optimal profit. By contrast, the optimal quality gap in the firm’s product line will always decrease when consumers have stronger over-purchase regret.

Finally, we analyze a model extension to relax the assumptions regarding how and when consumers’ uncertainty about their true quality valuation can be resolved by considering two popular strategies that a firm can employ. We discuss and compare the impact of a free-trial program that rids consumers of their uncertainty before purchase, and an exchange program that
eliminates consumers’ regret after purchase. We show that it may not be in the firm’s best interest to remove consumers’ anticipated regret.

The rest of the paper is organized as follows. Section 2 summarizes the relevant literature. Section 3 lays out our assumptions on the firm and consumers. Section 4 discusses the optimal product line with exogenous product quality and Section 5 analyzes the case with endogenous product quality. Section 6 presents two extensions and Section 7 concludes.

2. Literature Review

Our paper builds on the empirical and experimental evidence presented in the psychology literature that consumers may display regret during the decision-making process (e.g., Arkes et al. 2002, Kahneman and Miller 1986, Kahneman and Tversky 1982a, 1982b, Landman 1987). A vast body of literature has shown that individuals can anticipate potential future regret and may try to alleviate or avoid it when making decisions. Bell (1982) and Loomes and Sugden (1982) incorporate anticipated regret into economic decision theory and explain some choice anomalies that cannot be explained by standard expected-utility theory. Abundant evidence has shown that anticipated regret can significantly influence people’s decisions in many contexts, such as gambling (Zeelenberg 1999, Zeelenberg, Beattie, van der Pligt and de Vries 1996), negotiation (Larrick and Boles 1995), sexual behavior (Richard, van der Pligt and de Vries 1996), and committing traffic violations (Parker, Stradling and Manstead 1996). Specifically, many studies have shown that anticipated regret can significantly affect consumers’ purchase decisions, such as decisions on purchase timing (Cooke, Meyvis and Schwartz 2001), trade-off between brand name and price (Simonson 1992), lottery choice (Inman, Dyer and Jia 1997), choices about long-distance telephone service, personal computer purchases, and apartments to live in (Inman and Zeelenberg 1998). Our work explicitly incorporates consumers’ regret in the context of quality-differentiated products, and analyzes the firm’s optimal response to this important psychological factor.

Specifically, this paper contributes to the stream of literature on consumers’ anticipated regret and its implications for consumers and firms. For example, Nasiry and Popescu (2012) examine the case where consumers may regret their advance purchases if their valuation later turns out to be lower than the price, or regret their delayed purchases because of missing a price discount or encountering a stock-out. They show that while the former type of regret will lower the advance-
selling firm’s profit, the latter may benefit the firm, and that the firm should not advance sell when consumers’ propensity of anticipated regret is strong. They suggest that a profit-maximizing firm should provide partial refunds to partially mitigate but not fully eliminate consumers’ anticipated regret. Similarly, Diecidue, Rudy, and Tang (2012) also analyze consumers’ forward- or spot-purchase decision, and show that a consumer’s incorrect anticipation of her future type will reduce her expected surplus. Jiang, Narasimhan, and Turut (2017) study the impact of anticipated regret on a market entrant’s product quality and pricing decisions. They show that anticipated regret may create a win-win for the incumbent and the entrant by alleviating competition, which can foster the entrant’s product innovation. Engelbrecht-Wiggans and Katok (2008) and Filiz-Ozbay and Ozbay (2007) analytically and experimentally show that anticipated regret can lead to overbidding or underbidding in first-price sealed-bid auctions. Most of these studies analyze the effect of consumers’ anticipated regret on a firm selling or auctioning a single product. By contrast, our paper analyzes how anticipated regret can affect the optimal pricing and quality decisions of a firm selling a product line of vertically differentiated products. Syam, Krishnamurthy, and Hess (2008) consider consumers’ anticipated regret for customized versus standardized products. They show that the market share of customized products declines as consumers become more regret-averse, and consumers’ preference for the customized products can increase when more standard products are available because they will expect less regret from choosing the customized versions. Their paper studies horizontally differentiated products and focuses on consumers’ purchase decisions. In comparison, we consider a vertically differentiated product line and examine the firm’s optimal decisions about both pricing and quality in addition to the consumer-welfare implications.

Our paper also contributes to the growing literature that incorporates consumers’ behavioral considerations or cognitive constraints into the product-line design problem. This stream of research has analyzed the strategic impact of reference-group effects in luxury goods (Amaldoss and Jain 2008), the role of limited-edition products on a firm’s profits in the presence of reference-group effects among consumers (Amaldoss and Jain 2010), whether limited-edition products should be offered between competing brands (Balachander and Stock 2009), and price discrimination under loss aversion and state-contingent reference points for consumers (Carbajal and Ely 2016). Other research has focused on a firm’s product-line design when consumers have limited attention and thus neglect some relevant product aspects when making the purchase decision (Dahremoller and Fels 2015), when consumers need to incur costly deliberation to
uncover their valuations for quality (Guo and Zhang 2012), and when consumers have context-dependent preference (Orhun 2009). In addition, Zeelenberg et al. (1996) and Zeelenberg (1999) show that decision makers are regret-averse instead of risk-averse on many occasions, and that depending on the context, anticipated regret can promote either risk-averse or risk-seeking choices. Note that unlike risk aversion or the classical reference-dependent preference, which only depend on the probability distributions of possible prospects, consumer regret is also based on the outcomes of the consumers’ actions. For example, when consumers choose among different options in a product line, their tendency of regret from choosing a product of lower than desired quality can be systematically different from that of choosing the product which is more costly than desired. Our paper contributes to the behavioral product-line design literature by explicitly modeling the two distinctive types of regret, and assessing their differential impact on the firm’s product line. Furthermore, we analyze when and how the firm can alleviate or exploit consumers’ anticipated regret to achieve a higher profit.

3. Model

Consider a market in which a monopoly firm sells two vertically differentiated products: a high-end product (denoted by $H$) and a low-end one (denoted by $L$). The quality and the price of product $j$ ($j \in \{H, L\}$) are $q_j$ and $p_j$, respectively, where $q_H > q_L > 0$. Product quality is common knowledge to consumers (i.e., the firm’s products are search goods). The marginal cost of producing product $j$ is given by $c_j = C(q_j)$, where $C(\cdot)$ is a strictly convex function with $C(0) = 0$. This convex cost function captures the increasing difficulty of improving a product’s quality. We assume that the production cost has a constant return to scale, i.e., given product quality the firm’s marginal cost is a constant with respect to production quantity.

There is a unit mass of consumers in the market, each of whom has a unit demand for the product category (i.e., each consumer buys at most one product). Consumer $i$’s benefit from using a product with quality $q_j$ is given by $\theta_i q_j$, where $\theta_i$ reflects this consumer’s valuation for product quality. Consumers are heterogeneous in their quality valuation, which can be either high ($\theta_i = \bar{\theta}$) or low ($\theta_i = \theta < \bar{\theta}$).
One key characteristic of our model lies in consumers’ pre-purchase uncertainty about their true valuation of the product’s quality. In particular, we assume that consumers are uncertain about their actual valuation of the product (consumers’ own type) when they are making the purchase decisions because they have not yet fully experienced the product to learn how much they really need the product. Consumers know the distribution of their possible quality valuations, \( \tilde{\theta} \) or \( \theta \), and will learn their exact valuation only after purchasing and using the product. Let \( \rho_i \) denote the prior probability that consumer \( i \) will have a high quality valuation, i.e., she will find \( \theta_i = \tilde{\theta} \) after purchasing and experiencing the product (ex post). By contrast, \( (1 - \rho_i) \) captures her prior probability that \( \theta_i = \_ \) post purchase.

Furthermore, consumers differ in their prior beliefs about their true type (actual valuation of the product). An \( \alpha \) fraction of consumers has a low prior probability \( \rho_l \) of realizing \( \theta = \tilde{\theta} \) ex post, where \( 0 < \rho_l < 1 \). Their probability of realizing \( \theta = \theta \) is thus given by \( (1 - \rho_l) \). On the other hand, the remaining \( (1 - \alpha) \) fraction of consumers has a high prior probability \( \rho_h \) of realizing \( \theta = \tilde{\theta} \) ex post, where \( \rho_l < \rho_h < 1 \). These consumers realize \( \theta = \theta \) ex post with the probability of \( (1 - \rho_h) \). We label the former type of consumers as low-prior consumers with subscript \( l \) and the latter high-prior consumers with subscript \( h \). The expected valuations for the low-prior and high-prior consumers are thus given by \( \rho_l \theta + (1 - \rho_l)\tilde{\theta} \) and \( \rho_h \theta + (1 - \rho_h)\tilde{\theta} \), respectively, where high-prior consumers have a higher expected valuation ex ante.

It is worth highlighting that even low-prior consumers can turn out to have a high valuation ex post, and high-prior consumers can turn out to have a low quality valuation ex post. In aggregate, the total number of high-valuation consumers ex post is \( \alpha \rho_l + (1 - \alpha) \rho_h \), and that of the low-valuation consumers is \( \alpha (1 - \rho_l) + (1 - \alpha)(1 - \rho_h) \). Next, we detail the process through which true quality valuation is realized and discuss consumers’ expected utility before the purchase decision (See Figure 1).

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6 We consider products whose value is unclear to consumers without being extensively used. In other words, a simple product inspection and trial on the spot at the store cannot completely resolve consumers’ uncertainty in their valuation, e.g., a lawn mower.
After a consumer $i$ purchases and fully experiences product $j$, she learns her true valuation of quality $\theta_i \in \{\bar{\theta}, \underline{\theta}\}$, and her ex post utility is then given by $u_{ij} = \theta_i q_j - p_j$. If she had chosen the other product $k \neq j$, her ex post utility would have been $u_{ik} = \theta_i q_k - p_k$. If consumer $i$ ex post finds that $u_{ik} > u_{ij}$, i.e., she could have gained a higher utility if she had purchased product $k$ instead of product $j$ (which she chose), she will suffer from a disutility of her own sub-optimal purchase decision. We define this disutility as regret in our paper.

**Figure 1 Consumers’ valuation for quality before and after purchase**

Note that a consumer’s regret can be classified into two different types. The first type of regret arises from *under-purchase*, i.e., a consumer purchased the low-quality product ($L$) but ex post found out that she could have gained a higher utility if she had purchased the high-quality product ($u_{iL} < u_{iH}$). The second type of regret occurs from *over-purchase*, i.e., a consumer purchased the high-quality product ($H$) but ex post realized that she could have been better off if she had purchased the low-quality product ($u_{iH} < u_{iL}$). We refer to the former type of regret as “under-purchase regret” and the latter type as “over-purchase regret.” We use parameter $\gamma_{\text{under}} \in (0,1)$ to measure the consumer’s extent of under-purchase regret, and parameter $\gamma_{\text{over}} \in (0,1)$ to capture the consumer’s extent of over-purchase regret. Specifically, if consumer $i$ purchases the low-quality product ($L$), her disutility from under-purchase regret after her valuation realization $\theta_i$ is defined as

$$r_{iL}(\theta_i) = \gamma_{\text{under}} \cdot \max\{u_{iH}(\theta_i) - u_{iL}(\theta_i), 0\} = \gamma_{\text{under}} \cdot \max\{(\theta_i q_H - p_H) - (\theta_i q_L - p_L), 0\}. \quad (1)$$
If her true valuation turns out to be low (i.e., $\theta_i = \theta$), which matches the type of product she bought ($L$), she will not have any regret ($r_{iL} = \gamma_{under} \cdot 0 = 0$). If her true valuation turns out to be high (i.e., $\theta_i = \bar{\theta}$), then her regret will be $r_{iL} = \gamma_{under}(u_{iH} - u_{iL})$. Note that this consumer would never suffer from over-purchase regret because she has purchased the low-quality product.

On the other hand, if consumer $i$ purchases the high-quality product ($H$), her disutility from over-purchase regret after her valuation realization $\theta_i$ is defined as

$$r_{iH}(\theta_i) = \gamma_{over} \cdot \max\{u_{iL}(\theta_i) - u_{iH}(\theta_i) , 0\}$$

$$= \gamma_{over} \cdot \max\{(\theta_i q_L - p_L) - (\theta_i q_H - p_H), 0\}. \quad (2)$$

Similarly, if her true valuation turns out to be high, she will not have any regret ($r_{iH} = \gamma_{over} \cdot 0 = 0$). However, if her true valuation turns out to be low, her regret will be $r_{iL} = \gamma_{over}(u_{iL} - u_{iH})$. Note that this consumer would never suffer from under-purchase regret because she has purchased the high-quality product.

To summarize, after accounting for regret from the potentially suboptimal purchase decision, consumer $i$’s ex post utility of purchasing product $j$ with quality $q_j$ is given by

$$U_{ij} = u_{ij} - r_{ij} = \begin{cases} 
\theta_i q_L - p_L - \gamma_{under} \cdot \max\{0, u_{iH} - u_{iL}\}, & \text{if } j = L \text{ (purchased product } L) ; \\
\theta_i q_H - p_H - \gamma_{over} \cdot \max\{0, u_{iL} - u_{iH}\}, & \text{if } j = H \text{ (purchased product } H) . 
\end{cases} \quad (3)$$

The utility of consumers’ outside option is normalized to be 0. If a consumer chooses the outside option, she will not use this product and cannot resolve the uncertainty of her valuation $\theta_i$. Thus, she will not experience any form of regret.

4. **Exogenous product quality**

In this section, we analyze the firm’s optimal product-line design with the assumption that the quality levels of both versions of the products are exogenous. This assumption is reasonable when the firm cannot easily change the quality of its products in the short run. We start with the analysis in which consumers do not have any post-purchase regret, and then compare it with the case where post-purchase regret exists and influences the firm’s pricing decisions. To ensure that the firm provides the full product line by selling both high-end and low-end products, we make the following two assumptions throughout this section:
\[
\frac{\rho_l - \rho_h}{\rho_l \bar{\theta} + (1 - \rho_l) \bar{\theta} - \frac{C(q_L)}{L}} < \alpha < \frac{\rho_h - \rho_l}{\rho_l \bar{\theta} + (1 - \rho_h) \bar{\theta} - \frac{C(q_H) - C(q_L)}{q_H - q_L}},
\]

and

\[
\frac{C(q_H) - C(q_L)}{q_H - q_L} < \rho_l \bar{\theta} + (1 - \rho_l) \bar{\theta}.
\]

Inequality (4) suggests that the segment size of low-prior consumers (\(\alpha\)) should be intermediate. When \(\alpha\) is too low, the firm will sell a single product to target the high-prior consumers only. When \(\alpha\) is too high, the firm will target both high-prior and low-prior consumers with a single product. Inequality (5) suggests that the cost difference between product \(H\) and product \(L\) cannot be too high because a very high cost difference will make it unprofitable for the firm to produce product \(H\).

4.1. Benchmark case: No regret

In this section, we consider the benchmark case where consumers have no post-purchase regret, i.e., \(\gamma_{under} = \gamma_{over} = 0\). In this case, consumer \(i\)'s expected utility from purchasing product \(j\) is given by \(EU_{ij} = \rho_i(\bar{\theta}q_j - p_j) + (1 - \rho_i)(\bar{\theta}q_j - p_j)\), \(i \in \{h,l\}, j \in \{H,L\}\). The first expression in the equation gives consumer \(i\)'s utility if her valuation realization is \(\bar{\theta}\), while the second term gives her utility if her valuation realization is \(\bar{\theta}\). We characterize the equilibrium outcome in the following lemma, where we use superscript \(NR\) to indicate variables in the benchmark case, and superscript \(^*\) to denote the firm’s optimal strategies.

**Lemma 1.** When consumers have no post-purchase regret, the firm’s optimal prices are \(p^*_{LNR} = [\rho_l \bar{\theta} + (1 - \rho_l) \bar{\theta}]q_L\) and \(p^*_{HNR} = [\rho_h \bar{\theta} + (1 - \rho_h) \bar{\theta}]q_H - (\rho_h - \rho_l)(\bar{\theta} - \bar{\theta})q_L\). The firm’s profit is \(\pi^*_{NR} = \alpha \cdot \{(\rho_l \bar{\theta} + (1 - \rho_l) \bar{\theta})q_L - C(q_L)\} + (1 - \alpha) \cdot \{(\rho_h \bar{\theta} + (1 - \rho_h) \bar{\theta})q_H - (\rho_h - \rho_l)(\bar{\theta} - \bar{\theta})q_L - C(q_H)\}\). The low-prior consumers’ expected surplus is zero, and the high-prior consumers’ expected surplus is \((\rho_h - \rho_l)(\bar{\theta} - \bar{\theta})q_L\).

In the benchmark case, the firm captures all consumer surplus of low-prior consumers, whereas it leaves a positive amount of surplus to the high-prior consumers! This context with “high-valuation” (“low-valuation”) consumers often discussed in previous
research, then these results are consistent with the literature on vertical differentiation (e.g., Mussa and Rosen 1978, Desai 2001). Next, we discuss the firm’s product-line design in the presence of consumers’ anticipated regret.

4.2. The case of anticipated regret

When consumer \( i \) anticipates her potential post-purchase regret and considers the purchase decision, her expected utility of purchasing the low-quality product \( L \) is given by

\[
E[U_{iL}] = \rho_i \left( \overline{\theta} q_L - p_L - r_{iL}(\overline{\theta}) \right) + (1 - \rho_i) \left( \theta q_L - p_L - r_{iL}(\theta) \right),
\]

(6)

where

\[
r_{iL}(\overline{\theta}) = \gamma_{under} \cdot \max\{\left( \overline{\theta} q_H - p_H \right) - \left( \overline{\theta} q_L - p_L \right), 0\} = \gamma_{under}\left( \overline{\theta} q_H - p_H \right) - \left( \overline{\theta} q_L - p_L \right),
\]

and

\[
r_{iL}(\theta) = \gamma_{under} \cdot \max\{\left( \theta q_H - p_H \right) - \left( \theta q_L - p_L \right), 0\} = 0.
\]

The component of the first term in consumer \( i \)'s expected utility, \( \left( \theta q_L - p_L - r_{iL}(\overline{\theta}) \right) \), captures her utility of purchasing and using the low-quality product with a high valuation realization, \( \theta_i = \overline{\theta} \), as well as her disutility of under-purchase regret, \( r_{iL}(\overline{\theta}) \). By contrast, \( \left( \theta q_L - p_L - r_{iL}(\theta) \right) \) gives this consumer's utility of purchasing and using the low-quality product with a low valuation realization, \( \theta_i = \underline{\theta} \), as well as her (potential) disutility of under-purchase regret, \( r_{iL}(\underline{\theta}) \).\(^7\)

Similarly, consumer \( i \)'s expected utility of purchasing the high-quality product \( H \) is

\[
E[U_{iH}] = \rho_i \left( \overline{\theta} q_H - p_H - r_{iH}(\overline{\theta}) \right) + (1 - \rho_i) \left( \theta q_H - p_H - r_{iH}(\theta) \right),
\]

(7)

where

\[
r_{iH}(\overline{\theta}) = \gamma_{over} \cdot \max\{\left( \overline{\theta} q_L - p_L \right) - \left( \overline{\theta} q_H - p_H \right), 0\} = 0,
\]

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\(^7\) We can show that in equilibrium \( r_{lh}(\overline{\theta}) = r_{hk}(\overline{\theta}) = 0 \).
and
\[
\gamma_{IH}(\theta) = \gamma_{over} \cdot \max\{ (\theta q_L - p_L) - (\theta q_H - p_H), 0 \} = \gamma_{over} \cdot [(\theta q_L - p_L) - (\theta q_H - p_H)].
\]

The interpretation of different components of \( EU_{IH} \) is similar to that of \( EU_{IL} \), and is thus not repeated here. If consumer \( i \) buys the low-quality product (\( L \)), she will not experience any over-purchase regret and her expected under-purchase regret is:
\[
E[r_{iL}] = \rho_l \cdot \gamma_{under} [ (\bar{\theta} q_H - p_H) - (\bar{\theta} q_L - p_L) ].
\]  
(8)

Similarly, if consumer \( i \) buys the high-quality product (\( H \)), she will not experience any under-purchase regret and her expected over-purchase regret is:
\[
E[r_{iH}] = (1 - \rho_l) \cdot \gamma_{over} [ (\theta q_L - p_L) - (\theta q_H - p_H) ].
\]  
(9)

Recall that we focus on the case where the firm provides the entire product line. Given that the low-prior consumers have a lower expected valuation than their high-prior counterparts, the low-prior consumers will purchase the low-quality product, and the high-prior ones will purchase the high-quality product. To ensure both consumer segments purchase the product targeted at them, the firm needs to set prices such that both the low- and high-prior consumers’ individual rationality (\( IR \)) and incentive compatibility (\( IC \)) constraints are satisfied. They are given below:

\((IR_L)\): \[ \rho_l (\bar{\theta} q_L - p_L - \gamma_{under} \cdot r_{iL}(\bar{\theta})) + (1 - \rho_l) (\bar{\theta} q_L - p_L) \geq 0, \]
\((IR_H)\): \[ \rho_l (\bar{\theta} q_H - p_H) + (1 - \rho_l) (\theta q_H - p_H - \gamma_{over} \cdot r_{iH}(\theta)) \geq 0, \]
\((IC_L)\): \[ \rho_l (\bar{\theta} q_L - p_L - \gamma_{under} \cdot r_{iL}(\bar{\theta})) + (1 - \rho_l) (\theta q_L - p_L) \geq \rho_l (\bar{\theta} q_H - p_H) + (1 - \rho_l) (\theta q_H - p_H - \gamma_{over} \cdot r_{iH}(\theta)), \]
and
\((IC_H)\): \[ \rho_l (\bar{\theta} q_H - p_H) + (1 - \rho_l) (\bar{\theta} q_H - p_H - \gamma_{over} \cdot r_{iH}(\theta)) \geq \rho_l (\theta q_L - p_L - \gamma_{under} \cdot r_{iL}(\bar{\theta})) + (1 - \rho_l) (\theta q_L - p_L). \]

Solving the firm’s constrained optimization problem, we obtain its optimal prices, profits and consumer surplus when consumers have anticipated regret. To facilitate discussion, we define the regret function, \( G(\gamma_{under}, \gamma_{over}) \), as follows:
\[ G(\gamma_{\text{under}}, \gamma_{\text{over}}) \triangleq \frac{1}{\left(1 + \frac{1}{\gamma_{\text{under}}}\right) \left[1 + \frac{\rho_h}{(1 + \gamma_{\text{over}})(1 - \rho_h)}\right]} . \]  

Note that \( G(\gamma_{\text{under}}, \gamma_{\text{over}}) \) is strictly greater than 1 and it strictly increases in \( \gamma_{\text{under}} \) and \( \gamma_{\text{over}} \). Intuitively, \( G(\gamma_{\text{under}}, \gamma_{\text{over}}) \) captures the magnitude of consumer regret: consumers have a stronger level of regret when \( G(\gamma_{\text{under}}, \gamma_{\text{over}}) \) is larger.

The firm’s optimal prices depend on the levels of consumers’ anticipated regret and quality differentiation within its product line, and they are summarized as follows. When \( G(\gamma_{\text{under}}, \gamma_{\text{over}}) \leq \frac{a_L}{q_H} \), the firm’s optimal prices for the low- and the high-quality products are

\[ p_L^* = p_L^{NR*} - \gamma_{\text{under}}(1 + \gamma_{\text{over}}) \rho_L \frac{(1 - \rho_h)(\overline{\theta} - \theta)(q_H - q_L)}{1 + \gamma_{\text{under}} \rho_h + \gamma_{\text{over}}(1 - \rho_h)} \quad \text{and} \quad p_H^* = p_H^{NR*} + [(\gamma_{\text{under}} - \gamma_{\text{over}}) \rho_h - \gamma_{\text{under}}(1 + \gamma_{\text{over}}) \rho_L] \frac{(1 - \rho_h)(\overline{\theta} - \theta)(q_H - q_L)}{1 + \gamma_{\text{under}} \rho_h + \gamma_{\text{over}}(1 - \rho_h)} , \]

respectively.

When \( G(\gamma_{\text{under}}, \gamma_{\text{over}}) > \frac{a_L}{q_H} \), the firm’s optimal prices for the low- and the high-quality products are

\[ p_L^* = p_L^{NR*} - \rho_l(\overline{\theta} - \theta) \gamma_{\text{under}} \frac{(1 - \rho_h)(1 + \gamma_{\text{over}})q_H - [1 - \rho_l + \gamma_{\text{over}}(1 - \rho_h)]q_L}{1 + \gamma_{\text{under}} \rho_l + \gamma_{\text{over}}(1 - \rho_h)} \quad \text{and} \quad p_H^* = p_H^{NR*} - (\overline{\theta} - \theta) \frac{\gamma_{\text{over}}(1 - \rho_h)(\rho_l + \rho_l \gamma_{\text{under}})q_H - [(1 + \rho_l \gamma_{\text{under}})(\rho_h - \rho_l + \gamma_{\text{over}}(1 - \rho_h)) - (1 - \rho_h)^2 \gamma_{\text{over}}]q_L}{1 + \gamma_{\text{under}} \rho_l + \gamma_{\text{over}}(1 - \rho_h)} , \]

respectively.

When consumers have anticipated regret, the firm’s profit is given by

\[ \pi^* = \alpha(p_L^* - C(q_L)) + (1 - \alpha)(p_H^* - C(q_H)). \]  

One key question that we want to address is how consumers’ anticipated regret influences the firm’s optimal pricing. Proposition 1 below answers this question.

**Proposition 1.** When consumers anticipate the potential post-purchase regret:

1. When the propensity of anticipated regret is low, i.e., \( G(\gamma_{\text{under}}, \gamma_{\text{over}}) \leq \frac{a_L}{q_H} \), the firm’s optimal price for the low-quality product, \( p_L^* \), decreases with either type of regret, i.e., \( \frac{\partial p_L^*}{\partial \gamma_{\text{under}}} < 0 \)
and \( \frac{\partial p_H^*}{\partial \gamma_{\text{over}}} < 0 \). The firm’s optimal price for the high-quality product, \( p_H^* \), decreases with \( \gamma_{\text{over}} \), i.e.,
\[
\frac{\partial p_H^*}{\partial \gamma_{\text{over}}} < 0, \text{ but it may increase with } \gamma_{\text{under}}.
\]

(2) When the propensity of anticipated regret is high, i.e., \( G(\gamma_{\text{under}}, \gamma_{\text{over}}) > \frac{q_L}{q_H} \), the firm’s optimal prices, \( p_L^* \) and \( p_H^* \), decrease with either type of regret, i.e.,
\[
\frac{\partial p_L^*}{\partial \gamma_{\text{under}}} < 0, \quad \frac{\partial p_H^*}{\partial \gamma_{\text{under}}} < 0, \quad \frac{\partial p_L^*}{\partial \gamma_{\text{over}}} < 0 \quad \text{and} \quad \frac{\partial p_H^*}{\partial \gamma_{\text{over}}} < 0.
\]

We discuss the intuition of the two parts of Proposition 1 separately. First, when the propensity of consumer regret is below a threshold (i.e., \( G(\gamma_{\text{under}}, \gamma_{\text{over}}) \leq \frac{q_L}{q_H} \)), the classic results of second-degree price discrimination on consumer surplus remain unchanged. The low-prior consumers receive zero expected surplus, and high-prior consumers receive a positive amount of expected surplus such that they weakly prefer the high-quality product (technically, constraints \((IR_l)\) and \((IC_h)\) are binding).

One might intuit that when the propensity of consumer regret becomes stronger (i.e., \( \gamma_{\text{under}} \) or \( \gamma_{\text{over}} \) increases), consumers will anticipate a higher disutility from regret, hence their willingness to pay for products will decrease. As a consequence, the firm needs to lower its prices in equilibrium. While this reasoning largely carries through in this context, the first part of Proposition 1 suggests an exception: the firm’s optimal price for the high-quality product may actually increase with \( \gamma_{\text{under}} \).

The intuition is as follows. In the case where \( G(\gamma_{\text{under}}, \gamma_{\text{over}}) \leq \frac{q_L}{q_H} \), high-prior consumers would receive a positive surplus because their expected valuation of the same product (even for the low-quality product) is higher than that of the low-prior consumers. The firm needs to leave high-prior consumers enough surplus to incentivize them to purchase the high-quality product. Note that Equation (8) indicates that consumer \( i \)’s expected disutility from under-purchase regret, \( E[r_{IL}] = \rho_i \cdot \gamma_{\text{under}} [(\bar{\theta} q_H - p_H) - (\bar{\theta} q_L - p_L)] \), is proportional to \( \rho_i \), her prior probability of having a high valuation of quality. Therefore, when consumers start to anticipate stronger under-purchase regret (i.e., \( \gamma_{\text{under}} \) is higher), the low-quality product becomes much less attractive to high-prior consumers than to the low-prior consumers because buying the low-quality product is more likely to result in under-purchase regret ex post for high-prior than the low-prior consumers:
\( \rho_h > \rho_l \). Therefore, when \( \gamma_{\text{under}} \) increases, the firm can leave less surplus to the high-prior consumers and charge a higher price for its high-quality product. In other words, anticipated under-purchase regret actually helps the firm to reduce the cannibalization problem.

It seems that, following a similar argument, when consumers have stronger over-purchase regret (\( \gamma_{\text{over}} \) increases), the firm can increase its price of the low-quality product, \( p_L^* \), to reap more profits from the low-prior consumers. However, the first part of Proposition 1 suggests otherwise: a stronger over-purchase regret always lowers \( p_L^* \). The intuition is that although an increase in \( \gamma_{\text{over}} \) will make the high-quality product less attractive to low-prior consumers, they are not willing to pay more for the low-quality product. This is because the low-prior consumers are already left with zero surplus in equilibrium. In other words, it is always the \((IR_l)\) constraint, instead of the \((IC_l)\) constraint, that is binding for the low-prior consumers. Consequently, a stronger over-purchase regret does not allow the firm to raise the price of the low-quality product, because doing so will drive the low-prior consumers to choose the outside option of not buying anything.

Next we discuss the second part of Proposition 1, i.e., the case when the propensity of anticipated regret is high, i.e., \( G(\gamma_{\text{under}}, \gamma_{\text{over}}) > \frac{q_h}{q_H} \). Interestingly, in this case both consumer segments receive zero expected surplus (technically, constraints \((IR_l)\) and \((IR_h)\) are binding). This is in contrast to the classical finding in the vertical differentiation literature and that in our benchmark case in Section 4.1. The intuition is as follows. Previous discussion has shown that the expected surplus received by high-prior consumers decreases in the propensity of under-purchase regret, \( \gamma_{\text{under}} \). As a result, when \( \gamma_{\text{under}} \) is large enough, after taking into the disutility of under-purchase regret, the high-prior consumers may have a lower expected utility of the low-quality product \((L)\) than the low-prior consumers, i.e., \( EU_{hl} < EU_{ll} \). Therefore, the firm can now fully extract the surplus from high-prior consumers without worrying about them switching to the low-quality product.

When over-purchase regret \( \gamma_{\text{over}} \) is strong, the firm needs to reduce the price difference between the low- and the high-quality products to mitigate consumers’ over-purchase regret. This will increase the high-prior consumers’ expected under-purchase regret of buying the low-quality product more than that of the low-prior consumers. Therefore, the firm can fully extract the surplus from the high-prior consumers in equilibrium.
To summarize, when $G(\gamma_{under}, \gamma_{over}) > \frac{q_L}{q_H}$, even the high-prior consumers will receive zero surplus. Consequently, the firm will not be able to extract more surplus from high-prior consumers and cannot charge a higher $p_H$ when $\gamma_{under}$ increases. Therefore, the firm’s optimal prices for both the low- and the high-quality product, $p_H^*$ and $p_L^*$, always decrease with $\gamma_{under}$ and $\gamma_{over}$.

After discussing how consumers’ anticipated regret affects the firm’s optimal prices, we next examine how the firm’s profit will be influenced by anticipated regret. One might intuit that when consumers exhibit stronger anticipated regret, i.e., when $\gamma_{over}$ or $\gamma_{under}$ are higher, their expected utility from any purchase will decrease. As a result, the firm has to lower the prices and its profit will decrease. Interestingly, Proposition 2 below shows that this conventional wisdom may be flawed and that one type of anticipated regret can benefit the firm.

**Proposition 2.** (1) The firm’s profit increases with consumers’ propensity of under-purchase regret, $\gamma_{under}$, if and only if $G(\gamma_{under}, \gamma_{over}) \leq \frac{q_L}{q_H}$ and $\gamma_{over} < \frac{(1-\alpha)\rho_h-\rho_l}{\rho_l}$. (2) The firm’s profit always decreases with consumers’ propensity of over-purchase regret, $\gamma_{over}$.

Proposition 2 states that under-purchase regret may benefit the firm, but over-purchase regret always hurts the firm. Under-purchase regret may benefit the firm because it can lead to a higher price of the high-quality product, $p_H^*$, when $G(\gamma_{under}, \gamma_{over}) \leq \frac{q_L}{q_H}$. Moreover, the condition that over-purchase regret is not exceedingly high, $\gamma_{over} < \frac{(1-\alpha)\rho_h-\rho_l}{\rho_l}$, is also required. Recall that Proposition 1 shows that a stronger under-purchase regret (an increase in $\gamma_{under}$) always negatively affects the firm’s profit from the low-prior consumers, i.e., $\frac{\partial p_l^*}{\partial \gamma_{under}} < 0$. However, how a higher $\gamma_{under}$ affects the profit from high-prior consumers is ambiguous. On the one hand, a higher $\gamma_{under}$ means that the low-quality product becomes less appealing to high-prior consumers, who would suffer a greater under-purchase regret when considering the low-quality alternative. This consideration gives the firm some scope of raising the price on the high-quality product purchased by the high-prior consumers. On the other hand, an increase in $\gamma_{under}$ can also have a negative effect on the high-quality product’s price, $p_H$, and thus the profit from high-prior consumers. This occurs because as $\gamma_{under}$ increases, the low-prior consumers will expect a higher level of under-purchase regret by selecting the low-quality product, so the firm needs to reduce $p_L$. 
This reduction in $p_L$ leads to a greater expected over-purchase regret by high-prior consumers if they buy the high-end product. Mathematically, it can be seen as $E[r_{IH}] = (1 - \rho_l) \cdot \gamma_{over} \cdot [(\theta q_L - p_L) - (\theta q_H - p_H)]$ increases when $p_L$ decreases. In response, the firm has incentives to lower the high-quality product’s price, $p_H$.

Eventually, whether the firm will raise or lower the price $p_H$ depends on the trade-off between the two competing forces. If the propensity of over-purchase regret ($\gamma_{over}$) is high, the firm needs to lower $p_H$ dramatically when facing a higher $\gamma_{under}$ (technically, $\frac{\partial (\frac{\partial p_H^*}{\partial \gamma_{under}})}{\partial \gamma_{over}} < 0$). When $\gamma_{over}$ is low, the firm does not need to lower $p_H$ much when $\gamma_{under}$ increases. Therefore, if $\gamma_{over}$ is below a threshold, i.e., $\gamma_{over} < \frac{(1-\alpha)\rho_L - \rho_I}{\rho_I}$, the positive effect on the price from high-prior consumers due to a higher $\gamma_{under}$ dominates the negative effect. Therefore, the firm can benefit from consumers’ stronger under-purchase regret. By contrast, stronger over-purchase regret will always reduce the firm’s profit because the firm needs to lower both $p_L^*$ and $p_H^*$, as shown in Proposition 1.

After analyzing the impact of the two distinctive types of anticipated regret on the firm’s profits, we next discuss whether the firm can be more profitable in the presence of anticipated regret than in the absence of it. We answer this question in the next two Corollaries. Define $\Delta \pi^* = \pi^* - \pi_{NR}^*$ as the change in the firm’s optimal profit due to anticipated regret. We evaluate whether this profit difference can be positive.

**Corollary 1.** The firm’s profit is higher in the presence of consumers’ anticipated regret, i.e., $\Delta \pi^* > 0$ if and only if $\gamma_{over} < \frac{q_H[(1-\alpha)\rho_h - \rho_I]}{(1-\rho_h)[q_L \rho_I + \rho_h(1-\alpha)(q_H - q_L)]}$ and $\gamma_{under} \in (\gamma_{under}, \bar{\gamma}_{under})$.\(^8\)

**Corollary 2.** If the propensities of the two types of regret are equal, i.e., $\gamma_{under} = \gamma_{over} > 0$, the firm’s profit is lower than that in the benchmark case without anticipated regret.

\[^8\] $\gamma_{under} = \frac{(1-\alpha)\rho_I}{(1-\alpha)\rho_h - (1+\gamma_{over})\rho_I}$ and $\gamma_{over} = \left\{\begin{array}{ll}
\frac{1-\alpha}{\rho_I} \cdot \frac{q_H[(1-\alpha)\rho_h - q_L(1-\alpha)\rho_I]}{(1-\rho_h)[q_L \rho_I + \rho_h(1-\alpha)(q_H - q_L)]}, & \text{if } \gamma_{over} \geq \frac{(1-\alpha)q_H(1+\rho_h)q_L(1-2\alpha+\rho_h)-aq_H(1-\rho_h)}{(q_H-q_L)(1-\rho_h)[\rho_I + (1-\alpha)\rho_h]};
1, & \text{if } \gamma_{over} < \frac{(1-\alpha)q_H(1+\rho_h)q_L(1-2\alpha+\rho_h)-aq_H(1-\rho_h)}{(q_H-q_L)(1-\rho_h)[\rho_I + (1-\alpha)\rho_h]}.
\right.$
Corollary 1 shows that the firm can benefit from the consumer’s anticipated regret compared to the benchmark case: its profit is higher when the level of over-purchase regret is low and the level of under-purchase regret is moderate. In this case, the profit improvement from the higher price of the high-quality product more than offsets the revenue loss from the low-quality product. In comparison, Corollary 2 shows that when consumers’ propensities of over-purchase regret and under-purchase regret are equal in size, the firm’s profit is always lower than the case without anticipate regret. In this case, even if the under-purchase regret has a positive effect on the firm’s profit, it will be offset by the negative effect of an equal-sized over-purchase regret propensity. Put differently, the necessary condition for the firm to benefit from consumers’ anticipated regret is for under-purchase regret to be greater than over-purchase regret: $\gamma_{\text{under}} > \gamma_{\text{over}}$. Therefore, the firm has incentives to alleviate the consumers’ over-purchase regret, but at the same time it may want to reinforce consumers’ under-purchase regret.

In practice, the firm can utilize different strategies to influence consumers’ over-purchase and under-purchase regret. For example, the firm can choose which product (the high-quality product or the low-quality one) to highlight as the default option. Classic economic theory predicts that the choice of the default product will not affect consumers’ choices and thus the firm’s profit. However, the decision-making literature has consistently reported that by choosing the default choice, consumers tend to regret less ex post compared to selecting the non-default options (e.g., Kahneman and Miller 1986, Kahneman and Tversky 1982a, 1982b, Simonson 1992). Therefore, the firm may want to set the high-quality product as the default option to magnify consumers’ under-purchase regret and weaken their over-purchase regret. Similarly, when a firm offers a product line, it should advertise the drawbacks of not choosing the high-quality product to induce the under-purchase regret. For instance, in Intel’s advertisement “Rewind regret,” a consumer regrets that he did not buy a computer with a high-performance Intel processor when his computer with a slow processor crashes. At the end of the ad, Intel reminds consumers “You can’t rewind regret.”9 This advertising message from Intel is consistent with the idea of highlighting consumers’ under-purchase regret.

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Up to this point, we have characterized the firm’s profits in the presence of anticipated regret and how the two types of regret influence its profits. One natural question is how consumers fare with anticipated regret. We answer this question in the following proposition.

**Proposition 3.** The expected consumer surplus decreases with \( \gamma_{\text{under}} \) and \( \gamma_{\text{over}} \).

Proposition 3 states that consumers are always worse off when their propensity of anticipated regret increases, i.e., when \( \gamma_{\text{under}} \) or \( \gamma_{\text{over}} \) are higher. Recall that consumers’ utility is given by

\[
U_{ij} = u_{ij} - r_{ij} = \begin{cases} 
\theta_i q_L - p_L - \gamma_{\text{under}} \cdot \max\{0, u_{iH} - u_{iL}\}, & \text{if } j = L \text{ (purchased product } L) \\
\theta_i q_H - p_H - \gamma_{\text{over}} \cdot \max\{0, u_{iL} - u_{iH}\}, & \text{if } j = H \text{ (purchased product } H) 
\end{cases}
\]

Intuitively, an increase in \( \gamma_{\text{under}} \) and \( \gamma_{\text{over}} \) will lead to a higher disutility from anticipated regret, thus it will reduce the consumer’s overall surplus in expectation. Note that this result is obtained when the product quality is exogenous. Analysis in Section 5.2 will show how the result will be different when the firm endogenously determines the product quality.

One interesting observation is that although consumers’ expected surplus decreases, their ex post utility may increase compared with their utility without anticipated regret. For example, consider a low-prior consumer whose valuation realization turns out to be low, i.e., \( \theta_i = \overline{\theta} \). She will not experience any post-purchase regret, so her ex post utility from using the low-quality product is \( \overline{\theta} q_L - p_L^* - r_L(\overline{\theta}) = \overline{\theta} q_L - p_L^* \). Given our previous discussion that the low-quality product’s price, \( p_L \), decreases in the intensity of consumers’ anticipated regret, this low-prior consumer’s ex post utility can be higher than the ex post utility in the absence of anticipated regret. Similarly, if a stronger intensity of consumer regret can lead to a lower price of the high-quality product, \( p_H^* \), then high-prior consumers can enjoy an increase in their ex post utility if their valuation realization turns out to be high, i.e., \( \theta_i = \overline{\theta} \).

In summary, we have analyzed the firm’s optimal product line pricing with consumers’ anticipated regret when the quality levels of both products are exogenously given. This exogenous quality assumption is more reasonable in the short run when the firm can adjust prices much more easily than it can adjust product qualities. Next, we examine the firm’s product-line design when quality levels can be endogenously determined. By doing so, we can understand the impact of anticipated regret on both pricing and quality design.
5. Endogenous Quality

In this section, we endogenize the firm’s quality decision and analyze its optimal product-line design. In this case, the firm chooses both prices and quality levels of the two products, \((q_H, p_H)\) and \((q_L, p_L)\), to maximize its profit. Similar to Section 4, we still focus on the parameter region where the firm provides the full product line by selling both high- and low-end products.

We first analyze the firm’s optimal product-line decisions in the absence of anticipated regret, and then present the case with anticipated regret.

5.1. Benchmark case: no anticipated regret

When consumers have no anticipated regret, their utilities are solely determined by their expected valuations for product quality, \([\rho_i \theta + (1 - \rho_i)\theta]q_j - p_j\), where \(i \in \{h, l\}\) and \(j \in \{H, L\}\). Taking that into account, the firm optimizes prices and quality levels of its product line. We summarize the equilibrium quality levels in the following lemma (superscript NR denotes the case without anticipated regret, and ** denotes the equilibrium outcome when quality is endogenous).

**Lemma 2.** The optimal levels of product quality, \(q_{H NR}^{**}\) and \(q_{L NR}^{**}\), are uniquely determined by

\[
C'(q_{H NR}^{**}) = \rho_h \bar{\theta} + (1 - \rho_h)\bar{\theta} \quad \text{and} \quad C'(q_{L NR}^{**}) = [\rho_l \bar{\theta} + (1 - \rho_l)\bar{\theta}] - \frac{1 - \alpha}{\alpha} \cdot (\rho_h - \rho_l)(\bar{\theta} - \bar{\theta}).
\]

Consistent with the existing literature on vertical differentiation, the high-end product’s quality is at the efficient level: the marginal cost of production with respect to its quality is equal to high-prior consumers’ expected valuation, i.e., \(C'(q_{H NR}^{**}) = \rho_h \bar{\theta} + (1 - \rho_h)\bar{\theta}\). This quality level increases in \(\rho_h\), the probability of realizing a high valuation by high-prior consumers, as well as \(\bar{\theta}\) and \(\bar{\theta}\), the specific valuations post purchase and usage. By contrast, the low-end product’s quality is distorted downward from its efficient level, i.e., \(C'(q_{L NR}^{**}) < \rho_l \bar{\theta} + (1 - \rho_l)\bar{\theta}\). The extent of this downward distortion increases in consumers’ heterogeneity, \((\rho_h - \rho_l)\) and \((\bar{\theta} - \bar{\theta})\), and decreases in the size of the low-prior consumers, \(\alpha\).

Next, we present the firm’s optimal product-line design in the presence of anticipated regret, and contrast that with the benchmark case without regret.
5.2. The case of anticipated regret

When consumers have anticipated regret, the firm’s optimal quality levels will be affected by consumers’ propensities of over-purchase and under-purchase regret, $\gamma_{over}$ and $\gamma_{under}$. In particular, if consumers have bought the low-quality product but their valuation realization turns out to be high ($\theta_l = \bar{\theta}$), then their regret disutility would be $r_L(\bar{\theta}) = \gamma_{under} \cdot [(\theta q_H - p_H) - (\theta q_L - p_L)]$. On the other hand, if consumers have bought the high-quality product but their valuation realization turns out to be low ($\theta_l = \theta$), then their regret disutility would be $r_{iH}(\theta) = \gamma_{over} \cdot [(\theta q_L - p_L) - (\theta q_H - p_H)]$. Solving the firm’s profit maximization problem leads to the following proposition ($q_{j**}$ denotes the firm’s optimal quality level of product $j$).

**Proposition 4.** (1) When $G(\gamma_{under}, \gamma_{over}) < \frac{q_{L**}}{q_{H**}}$ and $\gamma_{over} < \frac{(1-\alpha)\rho_h - \rho_l}{\rho_l}$, $q_{L**}$ decreases and $q_{H**}$ increases with $\gamma_{under}$. Otherwise, $q_{L**}$ increases and $q_{H**}$ decreases with $\gamma_{under}$. (2) $q_{L**}$ increases and $q_{H**}$ decreases with $\gamma_{over}$.

The first part of Proposition 4 shows that when the propensity of anticipated regret is relatively low, i.e., $G(\gamma_{under}, \gamma_{over}) < \frac{q_{L**}}{q_{H**}}$ and $\gamma_{over} < \frac{(1-\alpha)\rho_h - \rho_l}{\rho_l}$, the optimal quality of the low-end product, ($q_{L**}$), decreases but that of the high-end product, ($q_{H**}$), improves with $\gamma_{under}$. One interesting observation is that these conditions are the same as the conditions under which the firm’s optimal profit increases with $\gamma_{under}$ with exogenous quality (See Proposition 2). This is more than a mere coincidence. Intuitively, if the firm’s profit increases with $\gamma_{under}$ when product quality is exogenous, the firm benefits from consumers’ under-purchase regret. In this case, the firm has incentives to further increase the magnitude of consumer under-purchase regret. Holding everything else constant, consumers’ under-purchase regret will be more profound when the difference between $q_H$ and $q_L$ is larger. Therefore, when the firm optimally chooses its quality, it will increase the quality of high-end products and lower the quality of low-end products as $\gamma_{under}$ increases. This strategy will allow the firm to better take advantage of the positive impact of $\gamma_{under}$ on the IC constraint of the high-prior consumers. On the contrary, when $G(\gamma_{under}, \gamma_{over}) \geq \frac{q_{L**}}{q_{H**}}$ or $\gamma_{over} \geq \frac{(1-\alpha)\rho_h - \rho_l}{\rho_l}$, both types of regret are already strong. Thus the firm has incentives to mitigate
consumer regret. It achieves that by reducing the quality of high-end products and improving the quality of low-end products as $\gamma_{\text{under}}$ increases.

The second part of Proposition 4 states that as the propensity of consumers’ over-purchase regret increases, the firm will improve the quality of the low-end product but reduce that of the high-end product. The intuition is analogous to the discussion on the first part of this proposition. Recall that when the level of over-purchase regret increases, consumers are generally more averse toward buying the high-end product. In fact, when product quality is exogenous, the firm’s profit always decreases with $\gamma_{\text{over}}$. Holding everything else constant, consumers’ over-purchase regret will be less pronounced when the difference between $q_H$ and $q_L$ is smaller. Therefore, when the firm can optimally choose its product quality, it will increase $q_L$ and lower $q_H$ to lower the negative impact of consumers’ over-purchase regret.

A key managerial decision regarding a product line is the quality differentiation between the options within the line. Based on Proposition 4, we can directly obtain the following insights on the effect of anticipated regret on the quality differences between the high- and low-end products.

**Corollary 3.** The quality gap in the product line, $q_H^{**} - q_L^{**}$, increases with $\gamma_{\text{under}}$ when $G(\gamma_{\text{under}}, \gamma_{\text{over}}) < \frac{q_L^{**}}{q_H^{**}}$ and $\gamma_{\text{over}} < \frac{(1-\alpha)\rho_h - \rho_l}{\rho_l}$. Otherwise the quality gap in the product line decreases with $\gamma_{\text{under}}$. The quality gap in the product line always decreases with $\gamma_{\text{over}}$.

Finally, a comparison of profits between the case with anticipated regret and the case without leads to the following result.

**Corollary 4.** There exists $\gamma_{\text{over}}^{**} > 0$ such that when $\gamma_{\text{over}} < \gamma_{\text{over}}^{**}$ and $\gamma_{\text{under}} \in (\gamma_{\text{under}}^{**}, \gamma_{\text{under}}^{**})$, the firm’s profit with anticipated regret is higher than the profit without regret, $\pi_{NR^{**}}^{**}$. \(^{10}\)

Corollary 4 states that the presence of anticipated regret can benefit a firm under certain conditions. The intuition is that when the under-purchase regret $\gamma_{\text{under}}$ is moderate, high-prior consumers have fewer incentives to consider the low-end product. In other words, the firm’s cannibalization problem becomes less severe due to a more slack incentive compatibility constraint.

\(^{10}\) The values of $\gamma_{\text{under}}^{**}$ and $\gamma_{\text{under}}^{**}$ depend on $\gamma_{\text{over}}^{**}$, and they are given in the Appendix.
for the high-prior consumer segment. As a result, the firm can raise the price of the high-end product, $p_H$. Furthermore, the additional revenue from the high-prior consumers with moderate under-purchase regret, $\gamma_{\text{under}}$, outweighs revenue loss from a reduced price of the low-end product, $p_L$.

Note that this result is consistent with Corollary 1 in Section 4.2, where the firm’s quality is exogenously given. It reinforces the robustness of the result that the presence of consumers’ anticipated regret is not necessarily detrimental to a firm’s profits. In fact, with a relatively low over-purchase regret and a moderate under-purchase regret, the firm can further exploit the high-prior consumer segment and improve its overall profits.

Having considered how anticipated regret influences the firm’s profit, we next discuss how consumer surplus will be affected when the firm endogenously determines its product quality levels in the following proposition.

**Proposition 5.** When product quality is endogenously determined, there may exist $\hat{\gamma}_{\text{over}} \geq 0$ and $\hat{\gamma}_{\text{under}} \geq 0$, such that the expected consumer surplus when $\gamma_{\text{under}} = \hat{\gamma}_{\text{under}}$ and $\gamma_{\text{over}} = \hat{\gamma}_{\text{over}}$ is higher than that when $\gamma_{\text{under}} = \gamma_{\text{over}} = 0$. In such a case, the firm’s optimal quality for the low-end product is higher than the benchmark case, i.e., $q_L^{**} \geq q_L^{NR**}$.

One might reason that when consumers suffer the disutility from regret, they will be worse off than when they are immune to anticipated regret. Proposition 3 validates this argument when the product quality is exogenous. In stark contrast, Proposition 5 states that if the firm endogenously decides its product quality, consumers’ anticipated regret can actually lead to higher expected consumer surplus. The intuition hinges on the firm’s strategic quality decision in response to consumers’ anticipated regret. When $G(\gamma_{\text{under}}, \gamma_{\text{over}}) < \frac{q_L^*}{q_H}$, the expected surplus of low-prior consumers is zero, so the total expected consumer surplus is equal to the expected surplus of high-prior consumers. Note that high-prior consumers’ expected surplus is the same as the expected utility that they could have derived from buying the low-quality product, $(L)$, which increases with $q_L$. Therefore, when anticipated regret increases the firm’s optimal level of $q_L$, it is possible that high-prior consumers receive higher expected surplus than when they are free of regret, although they do suffer from the disutility of regret.
In summary, we have examined how consumers’ anticipated regret influences the firm’s optimal levels of product quality, and shown that anticipated regret can increase consumer surplus in expectation. The endogenous quality assumption is more appropriate for understanding the firm’s long-term decisions in response to consumers’ anticipated regret, as the firm can flexibly adjust the product quality in the long term. In the next section, we will examine how the firm may utilize two specific strategies, a free trial program and an exchange program, to alleviate the impact of consumer regret and earn a higher profit.

6. Extensions

In Sections 4 and 5, we have analyzed the firm’s optimal product-line design in the presence of consumers’ anticipated regret. The fundamental issue in this context is that consumers have uncertainty about their true valuations on product quality, and this uncertainty can only be fully resolved after purchasing the product and using it for an extended period of time. One key reason for why anticipated regret persists is that it is fairly costly to remove consumers’ uncertainty before purchase (think of product wear and tear during trials and inspections leading to significant costs for the firm, and time costs for the consumers). While a lot of product categories, such as automobiles, home appliances and even housing, are representative examples of this context, there are other product categories where the cost of removing consumers’ valuation uncertainty and thus their anticipated regret is significantly lower. For example, smaller consumer electronics products, such as Bluetooth headphones and E-readers, may not impose prohibitive costs for both the firm and consumers to rid consumers’ uncertainty. In this section, we analyze two distinct strategies that the firm can employ, a free trial program and an exchange program, to alleviate the impact of consumers’ anticipated regret. The key difference between the two programs is that the former takes care of consumers’ uncertainty prior to their purchase decision, while the latter occurs after consumers’ purchase and usage. To focus on the strategic impact of these two programs on consumers’ anticipated regret and simplify the analysis, we assume that product quality levels are exogenously given in this section. We start with analysis of a free trial program.

6.1. The case of a free trial

In this subsection, we analyze the case where the firm offers a free trial program to consumers prior to their purchase decision. We assume that it is costless to implement this program, and
consumers can get rid of their valuation uncertainty by the end of the free trial period. Specifically, once the firm allows its consumers to extensively try the product for free, consumers can learn about their true valuation, \( \theta \), before their purchase decisions and completely eliminate potential regret.

To maintain consistency with the main body of the paper, we still focus on the situation where the firm offers two products in equilibrium. The following condition ensures that:

\[
\frac{\theta - c(q_H)}{\bar{q} - c(q_L)} \leq \alpha \leq \frac{1}{\rho_H - \rho_l} \left[ \rho_H - \frac{\theta - c(q_H) - c(q_l)}{\bar{q} - c(q_H) - c(q_l)} \right].
\]

Intuitively, this condition on \( \alpha \) implies that when there are a moderate number of consumers in each valuation segment, it is in the firm’s best interest to target high- and low-prior consumers with two distinct products. Offering a single product priced at low-prior consumers’ willingness to pay to serve everyone but forgo the high profit margin of the high-prior consumers is not profitable, nor is it profitable to offer only the high-end product to high-prior consumers and completely forgo the revenue from low-prior consumers. Solving the firm’s optimization problem, we obtain the following equilibrium results. First, the high-quality product’s price is \( p_{H,FT}^* = \bar{\theta} q_H - \bar{\theta} q_L + \theta q_L \), and the low-quality product’s price is \( p_{L,FT}^* = \theta q_L \) (We use the subscript “FT” to indicate the free trial case.). Second, the firm’s profit is

\[
\pi_{FT}^* = [\alpha \rho_l + (1 - \alpha) \rho_l] \left( p_{H,FT}^* - c(q_H) \right) + [\alpha(1 - \rho_l) + (1 - \alpha)(1 - \rho_h)] \left( p_{L,FT}^* - c(q_L) \right).
\]

After trying the product, \( \alpha \rho_l + (1 - \alpha) \rho_l \) number of consumers, including both high- and low-prior consumers, will find \( \theta = \bar{\theta} \) and choose to purchase the high-quality product. The remaining \( \alpha(1 - \rho_l) + (1 - \alpha)(1 - \rho_h) \) number of consumers, again including both high- and low-prior consumers, will find \( \theta = \theta \) and choose to purchase the low-quality product. In comparison, when the firm does not provide free trials (Section 4), the number of consumers who purchase the high-quality product is \( (1 - \alpha) \), and all of them are high-prior consumers. On the other hand, all low-prior consumers, with a total number of \( \alpha \), will purchase the low-quality product. Furthermore, recall that the firm’s optimal profit without a free trial is \( \pi^* = (1 - \alpha) (p_H^* - c(q_H)) + \alpha (p_L^* - c(q_L)) \).

Let \( \Delta \pi_{FT}^* = \pi_{FT}^* - \pi^* \) denote the net change in the firm’s profit due to free trials in equilibrium. We can decompose the profit comparison as follows:
\[
\Delta \pi_{\text{FT}}^* = \frac{(1 - \alpha)(p_{l,\text{FT}} - p_h^*) + \alpha(p_{l,\text{FT}} - p_l^*) + [\alpha \rho_l - (1 - \alpha)(1 - \rho_h)]([p_{H,\text{FT}} - C(q_H)] - [p_{L,\text{FT}} - C(q_L)])}{\Delta \pi_{\text{FT},1}}
\]

Note that the free trial program described here has two effects on the firm’s equilibrium profit, \(\pi_{\text{FT}}^*\). On the one hand, the \textit{price change effect} of the free trial, captured by \(\Delta \pi_{\text{FT},1}^*\), measures how the firm’s optimal prices change from \(p_l^*\) and \(p_H^*\) to \(p_{l,\text{FT}}^*\) and \(p_{H,\text{FT}}^*\), respectively. This change occurs because consumers will learn their true valuation \(\theta\) from the free trial, and will adjust their willingness to pay accordingly. On the other hand, the \textit{demand shift effect}, denoted by \(\Delta \pi_{\text{FT},2}^*\), captures how the number of buyers for each product changes. Because the firm’s profit margin for the high-quality product is higher than it is for the low-quality product, i.e., \(\left(p_{H,\text{FT}}^* - C(q_H)\right) - \left(p_{L,\text{FT}}^* - C(q_L)\right) > 0\), the demand shift effect will lead to a higher \(\pi_{\text{FT}}^*\) if and only if more consumers end up purchasing the high-end product, \(q_H\), after the free trial, i.e., when \(\alpha \rho_l - (1 - \alpha)(1 - \rho_h) > 0\). Given that the firm’s optimal prices, \(p_{H,\text{FT}}^*\) and \(p_{L,\text{FT}}^*\), do not depend on \(\alpha\), \(\rho_l\) and \(\rho_h\), the demand shift effect is stronger when consumers are more likely to realize \(\theta = \bar{\theta}\) and purchase the high-quality product after free trial, i.e., when probabilities, \(\rho_l\) and \(\rho_h\), are high. The demand shift effect is also stronger when fewer consumers would purchase the higher-quality product without the free trial, i.e., \(\alpha\) is high.

Next, we discuss the price change effect. To disentangle it from the demand shift effect, we analyze a special case where the number of buyers for each product with the free trial is the same as that without the free trial by assuming \(\alpha \rho_l = (1 - \alpha)(1 - \rho_h)\), i.e., \(\alpha = \frac{1 - \rho_h}{1 - \rho_h + \rho_l}\). In other words, the total number of consumers who purchase the high-end (low-end) product is the same regardless of whether the free trial is offered or not. This assumption prevents the demand shift effect from influencing the firm’s profit at all, i.e., \(\Delta \pi_{\text{FT},2}^* = 0\). As a result, the profit change will be exclusively due to the price change effect. Under this assumption, we can show that
\[
\Delta \pi_{\text{FT}}^* = \Delta \pi_{\text{FT},1}^* = \frac{\rho_l(\bar{\theta} - \theta)(q_H - q_L)(1 - \rho_h)}{1 + \rho_l - \rho_h} - \rho_l q_H(\bar{\theta} - \theta) - \Delta \pi^* ,
\]

where \(\Delta \pi^* = \pi^* - \pi_{NR}^*\) is the difference between the profit with anticipated regret (but without the free trial program) and the profit without anticipated regret (See Section 4.2).

The following proposition answers the key question regarding the profitability of the free trial program.
**Proposition 6.** The firm can increase its profit by providing the free trial if and only if the quality differentiation is sufficiently high in the product line: $q_H > K^*q_L$ ($K^* = K^*(\alpha, \rho_H, \rho_L, \gamma_{under}, \gamma_{under}) > 0$ is a constant and is defined in the appendix).

This proposition lays out the condition for a free trial program to be more profitable: a sufficient quality differentiation between the two options within the product line. The intuition is that with the help of a free trial, consumer $i$ will evaluate the product based on her true valuation for quality $\theta_i$, instead of her expected valuation, $\rho_i\overline{\theta} + (1 - \rho_i)\overline{\theta}$. Consumers who eventually buy the high-end product are willing to pay more compared to the situation without the free trial, because their decision is now based on their realization, $\theta = \overline{\theta}$, which is greater than their expected valuation for quality, $\rho_H\overline{\theta} + (1 - \rho_H)\overline{\theta}$. As a result, the firm can set the price for the high-quality product, $p_{H,FT}$, higher than the price without the free trial, $p_H$, i.e., $p_{H,FT} > p_H$. As a result, the free trial program benefits the firm more when the high-end product’s quality, $q_H$, is higher. In contrast, the free trial reduces the low-end product buyers’ marginal utility of product quality from $\rho_l\overline{\theta} + (1 - \rho_l)\overline{\theta}$ to $\overline{\theta}$. In this case, the drop in these consumers’ willingness to pay is greater when $q_L$ is higher. Overall, when $q_H > K^*q_L$, the increased profit from the high-end product buyers more than offsets the loss from the low-end product buyers. Consequently, a free trial program improves the firm’s profit when the quality differentiation between the two products is beyond a threshold.

One direct implication of Proposition 6 is that even absent any cost consideration, it may be in the firm’s best interest not to offer a free trial. In fact, the firm benefits from consumers’ uncertainty about their true quality valuation when the two versions of the products are relatively homogenous in quality. In this case, the additional revenue from the low-end product buyers’ increased willingness to pay outweighs the loss from the high-end product buyers.

### 6.2. The case of an exchange market

In this subsection, we consider how an exchange market can alleviate consumers’ regret and affects the firm’s pricing strategy of its product line. In particular, we analyze the situation where post-purchase, consumers have access to a secondary market that allows them to exchange their products with other consumers in order to rid their post-purchase regret. Note that in contrast to the pre-purchase free trial program discussed in Section 6.1, this exchange occurs after purchase.
For example, think of a consumer who bought the high-quality product but ended up with a low valuation realization of $\theta$. With the help of this exchange market, she can reduce her over-purchase regret by exchanging her high-quality product for a low-quality product, and receiving some compensation. Her exchange partner will be a consumer who has bought a low-end product, but realized a high valuation ($\theta_i = \overline{\theta}$) and is thus experiencing under-purchase regret.

To focus on the impact of this post-purchase mechanism on consumers’ regret, we simplify the analysis by assuming that the exchange platform is owned by the firm. Consumers are fully aware of the existence of the exchange market before purchase. In this case, there is one additional stage to the game outlined in the main model: After consumers buy the product and find out their valuation, $\theta$, they may use the exchange market and trade their purchases with each other. In particular, those who bought a low-quality product can pay the exchange price $p_{EM}$ to consumers who bought a high-quality one (the subscript “EM” stands for “exchange market”), and exchange products with them. The platform charges a transaction fee, $e_L (e_H)$, to consumers who exchange a low-quality (high-quality) product for a high-quality (low-quality) one. Note that $\alpha\rho_l$ fraction of consumers bought the low-quality product but later realized a high valuation, $\overline{\theta}$. These consumers want to exchange what they bought for the high-quality product. By contrast, $(1 - \alpha)(1 - \rho_h)$ fraction of consumers bought the high-quality product but ended up with a low valuation, $\overline{\theta}$, so they want to exchange what they bought for the low-quality product. Henceforth we refer to the former type of consumers as upgrading consumers and the latter type as downgrading consumers. To ensure that the exchange market clears, we assume that the two cohorts of consumers are of equal size: $\alpha\rho_l = (1 - \alpha)(1 - \rho_h)$ (i.e., $\alpha = \frac{1 - \rho_h}{1 - \rho_h + \rho_l}$).\textsuperscript{11}

With the exchange market, consumer $i$’s expected utility of purchasing the low-quality product is $E[U_{iL}^E] = \rho_i[\theta q_H - (p_L + p_{EM} + e_L) - \gamma_{under} \cdot \max\{p_{EM} + p_L + e_L - p_H, 0\}] + (1 - \rho_i) \cdot (\theta q_L - p_L)$ (superscript “E” denotes for consumers’ utility with the exchange market). The second component in the first term, $-(p_L + p_{EM} + e_L)$, reflects the total expenditure in the case where consumer $i$’s valuation realization is $\theta_i = \overline{\theta}$: she has initially paid price $p_L$ to obtain the low-

\textsuperscript{11} Otherwise, the group of consumers whose number is lower enjoys more bargaining power than their counterparts. In the extreme case where they enjoy the total bargaining power, they can extract all surplus from the other group who intend to exchange their mismatched products. The analysis of this case is available upon request.
quality product, and subsequently paid exchange fees $p_{EM}$ to a downgrading consumer and $e_L$ to the exchange platform. The third component, $\gamma_{under} \cdot \max\{p_{EM} + p_L + e_L - p_H, 0\}$, captures this consumer’s under-purchase regret because she potentially paid more than $p_H$, the price that she would be paying had she known her true type. The second term, $(1 - \rho_i)(\theta q_L - p_L)$, gives her expected utility of using the low-end product and finding her realized valuation type to be low, $\theta_i = \theta$. On the other hand, consumer $i$’s expected utility of buying the high-quality product is $E[U_{ih}^E] = \rho_i(\overline{\theta} q_H - p_H) + (1 - \rho_i)[\theta q_L - (p_H - p_{EM} + e_H) - \gamma_{over} \cdot \max\{p_H - p_L - p_{EM} + e_H, 0\}]$. In this case, she would receive the payment $p_{EM}$ from an upgrading consumer and pay a transaction fee, $e_H$, to the platform conditional on realizing a low valuation and engaging in the exchange. Once the event occurs, she would experience the over-purchase regret of $\gamma_{over} \cdot \max\{p_H - p_L - p_{EM} + e_H, 0\}$. The term $\rho_i(\overline{\theta} q_H - p_H)$ is simply her expected utility if her valuation realization, $\overline{\theta}$, matches the high-end product she bought.

The firm chooses the exchange price, $p_{EM}$, transaction fees, $e_H$ and $e_L$, together with the product prices, $p_H$ and $p_L$, to maximize its total profit from selling the product and facilitating the post purchase exchange platform. The following lemma summarizes the platform’s optimal exchange price and transaction fees in the exchange market.

**Lemma 3.** The optimal fees, $p_{EM}^*$, $e_H^*$ and $e_L^*$, satisfy $p_{EM}^* + e_L^* = \overline{\theta}(q_H - q_L)$ and $p_{EM}^* - e_H^* = \overline{\theta}(q_H - q_L)$. The firm’s profit from the exchange market alone is $\frac{(1 - \rho_H)\rho_i(\overline{\theta} - \theta)(q_H - q_L)}{1 - \rho_H + \rho_i}$, and it extracts all of the consumer surplus from exchanging their products, i.e., the utility and post-purchase regret of consumers remain unchanged after the exchanges.

One might intuit that with the exchange market, the upgrading consumers will be better off. After all, they find their valuation for quality to be high, and can get their desired high-quality product by getting rid of their low-end ones. Similarly, the downgrading consumers may also be better off because after realizing their valuation for quality to be low, they are able to get back some money from exchanging away the “unnecessary” high-end product. In this sense, both upgrading and downgrading consumers’ regret can be at least partially alleviated. However, when the platform is run by the firm, it will set the transaction fees high enough to extract all of the
surplus that consumers can gain from exchanging products. Consequently, consumers’ regret and net utility will remain unchanged after the exchanges.\(^{12}\)

By either allowing consumers to try for free before purchase or facilitating their exchanges after purchase, the firm can resolve consumers’ ex post mismatch between their purchase choice and their realized valuation for quality. In general, these strategies will be beneficial for the firm if there are no significant costs associated with them. For example, absent cost consideration, the exchange-market strategy always strictly increases the firm’s total profit (compared to the profit in Section 4.2), i.e., \(\pi^*_{EM} > \pi^*\). One natural question is the relative effectiveness between these two approaches. Next, we compare the firm’s total profit with the free trial and its profit with the exchange market in the following proposition (Recall that \(\alpha = \frac{1-\rho_h}{1-\rho_h+\rho_l}\) so that the post-purchase exchange market clears itself.).

**Proposition 7.** The exchange market is more profitable than the free trial, i.e., \(\pi^*_{EM} > \pi^*_{FT}\), if and only if \(\Delta\pi^* + \rho_l q_L (\bar{\theta} - \theta) > 0\), where \(\Delta\pi^* = \pi^* - \pi^{NR*}\), defined in Section 4.2, is the difference between the profit with anticipated regret and the profit without it.

Proposition 7 identifies the condition under which the exchange market owned by the firm dominates the free trial. To see the intuition, note that there are two important ways that the exchange market differs from the free trial. First, under the exchange market, the firm will set the transaction price and fees, \(p^*_{EM}, e^*_H\) and \(e^*_L\), such that consumers end up with the same level of regret as they have when no exchange market is available. Therefore, consumers’ anticipated regret will still affect the firm’s profit by the amount of \(\Delta\pi^*\), the same as when the exchange platform is unavailable. In contrast, under the free trial, consumers find out their valuation \(\theta\) before buying the product so they are free of regret. As a result, consumers’ anticipated regret would not affect the firm’s profit. Second, under the exchange market, consumers’ uncertainty of \(\theta\) remains at the moment of purchase. Because exchanging the product on the platform post purchase will not influence consumers’ utility, the firm does not need to give buyers of high-quality products more surplus. In contrast, the free trial resolves the consumers’ uncertainty of \(\theta\) so the difference in valuation for quality for the two groups of buyers will increase to \(\bar{\theta} - \theta\), from \((\rho_h - \rho_l)(\bar{\theta} - \theta)\)

\(^{12}\) If the exchange platform is owned by a profit-maximizing third-party rather than the firm itself, our results will remain the same, except that the third-party, instead of the firm, will earn all the profit from the exchange market.
when free trials are unavailable. Therefore, the firm needs to leave the high-quality product buyers with more surplus to prevent them from buying the low-quality product. This action leads to a loss of $\rho_lq_L(\bar{\theta} - \theta)$ in the firm’s profit. Hence, the exchange market strategy yields a higher profit for the firm than the free trial when the consumers’ anticipated regret does not reduce the firm’s profit too much, i.e., $\Delta\pi^*_r$ is above a threshold, or when the firm has to give too much extra surplus to buyers of high-quality products under the free trial, i.e., $\rho_lq_L(\bar{\theta} - \theta)$ is sufficiently high.

7. Conclusion

Anticipated regret is a well-documented consumer behavioral pattern. The objective of this paper is to shed light on how consumers’ anticipated regret influences a firm’s product-line design. To this end, we propose a model in which consumers have uncertainty about their valuation for product quality before purchase, and they exhibit post purchase regret if the product they bought did not match their realized preferred type. Consumers differ in their probabilities of having a high valuation for product quality, and the firm takes that into account when designing two vertically differentiated products. Our analysis offers some insights with regard to the firm’s optimal pricing and quality decisions in the presence of two different types of consumer regret, and provides a better understanding of a few questions of managerial significance.

First, how does consumers’ anticipated regret affect the firm’s pricing decisions in its product line? Our analysis shows that when consumers have sufficiently strong anticipated regret, the firm should reduce the entire product line’s prices to account for consumers’ lowered willingness to pay. Even when consumers’ anticipated regret is moderate, the firm should decrease its low-end product price. However, the firm may increase its high-end product’s price if consumers display a stronger under-purchase regret. The intuition is that when consumers have a stronger under-purchase regret, the low-quality product becomes much less attractive to consumers who are more likely to have a high valuation for quality ex post. Should these consumers buy the low-end product, they are more likely to experience under-purchase regret ex post. As a result, with a stronger under-purchase regret, the firm can leave less surplus to these consumers and charge a higher price for its high-quality product. Put differently, anticipated under-purchase regret helps the firm to reduce the cannibalization problem.
Second, how does consumers’ anticipated regret influence the firm’s quality decisions? Our results suggest that when both types of regret are relatively strong, the firm should reduce the quality of its high-end product and improve the quality of its low-end product. Narrowing the quality gap within its product line helps the firm mitigate the negative impact of either type of consumer regret. On the other hand, when both types of regret are moderate and consumers display a relatively stronger under-purchase regret, the firm should increase the quality of its high-end product and reduce the quality of its low-end product, making the quality differentiation in its product line stronger. Doing so allows the firm to better exploit the surplus of the high-end product buyers.

Third, what happens to consumer welfare in the presence of anticipated regret? When the quality of both products is exogenously given, compared to the case without anticipated regret, consumers’ welfare always decreases when they have a stronger under-purchase or over-purchase regret. Intuitively, an increase in either type of anticipated regret leads to a higher disutility in consumers’ expected utility before purchase, and it results in a drop in consumers’ overall surplus in expectation. However, we observe the opposite pattern when the firm endogenously determines the quality of its product line. In this case, consumers can be better off in the presence of anticipated regret. The driving factor is the improved quality of the low-end product, which becomes an improved alternative for the high-end consumers, forcing the firm to give them more surplus in equilibrium. As a result, the total consumer welfare can be improved despite the disutility from anticipated regret. It is worth highlighting that although regret is generally considered as a negative emotion, consumers may still benefit from it after accounting for the firm’s strategic response to this type of emotion.

Fourth, should the firm adopt strategies to remove consumers’ anticipated regret? The answer is “not always.” Our analysis shows that the firm is not necessarily worse off when consumers anticipate possible post-purchase regret. While the firm’s profit always decreases in consumers’ anticipated over-purchase regret, its profit may increase in consumers’ anticipated under-purchase regret. This result holds both when quality is exogenous and when quality is endogenized. The key reason is that a higher anticipated under-purchase regret strengthens high-end product buyers’ aversion toward the low-end product for the fear of a stronger post-purchase regret, and as a result, allows the firm to charge a higher price for the high-end product. Furthermore, we discuss and
compare the impact of two strategies that can eliminate consumers’ anticipated regret, the free trial before purchase and the exchange market after purchase. Again, we find that absent cost considerations, the firm may not be better off by using these programs to remove consumers’ anticipated regret. These results demonstrate how a negative consumer emotion such as regret can potentially benefit a firm’s profits in its product-line offering.

To focus on the strategic impact of consumers’ anticipated regret on a firm’s product-line design, we have made several simplifying assumptions in our model. For example, we examined a static model in which consumers’ uncertainty always exists before purchase (unless the firm offers a free trial). Future research can analyze a dynamic setting where consumers’ learning influences their post-purchase regret. We focused on a monopoly firm’s pricing and quality decisions within a product line; new insights can be obtained by studying the impact of consumers’ regret in competition. Another avenue for future research is to experimentally validate the theoretical predictions of our model (e.g., Amaldoss and Shin 2011, Ho, Lim and Cui 2010).
Reference

Zeelenberg M (1999) Anticipated refret, expected feedback, and behavioural decision making. J.
Appendix

**Figure A1: Levels of Reported Regret**

<table>
<thead>
<tr>
<th>Regret level</th>
<th>Fitness tracker</th>
<th>TV</th>
<th>Phone</th>
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<tr>
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<td>3.36</td>
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</tr>
<tr>
<td>3</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3.55</td>
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<td></td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1: Minimum Regret  
5: Maximum Regret

**Proofs:**

**Proof of Lemma 1.**

When consumers do not anticipate post-purchase regret, it is straightforward to see that the individual rationality (IR) constraint is binding for low-prior consumers, and the incentive compatibility (IC) constraint is binding is for high-prior consumers. Therefore, the firm’s optimal prices, \( p_{H}^{NR*} \) and \( p_{L}^{NR*} \), must satisfy:

\[
(IR_{L}): \rho_{L}(\overline{\theta}q_{L} - p_{L}^{NR*}) + (1 - \rho_{L})(\overline{\theta}q_{L} - p_{L}^{NR*}) = 0,
\]

and

\[
(IC_{H}): \rho_{H}(\overline{\theta}q_{H} - p_{H}^{NR*}) + (1 - \rho_{H})(\overline{\theta}q_{H} - p_{H}^{NR*}) = \rho_{H}(\overline{\theta}q_{L} - p_{L}^{NR*}) + (1 - \rho_{H})(\overline{\theta}q_{L} - p_{L}^{NR*}).
\]

Rearranging these two equations yields \( p_{L}^{NR*} = \left[ \rho_{L}\overline{\theta} + (1 - \rho_{L})\overline{\theta} \right]q_{L} \) and \( p_{H}^{NR*} = \left[ \rho_{H}\overline{\theta} + (1 - \rho_{H})\overline{\theta} \right]q_{H} - (\rho_{H} - \rho_{L})\overline{\theta}q_{L} \). The firm’s profit is given by \( \pi^{NR*} = (1 - \alpha)(p_{H}^{NR*} - C(q_H)) + \)
\[ \alpha [p_l^{NR*} - C(q_L)] = \alpha \cdot \{ [\rho_l \bar{\theta} + (1 - \rho_l)\theta]q_L - C(q_L) \} + (1 - \alpha) \cdot \{ [\rho_h \bar{\theta} + (1 - \rho_h)\theta]q_H - (\rho_h - \rho_h)(\bar{\theta} - \theta)q_L - C(q_H) \}. \]

**Proof of Proposition 1.**

Depending on which constraints are binding for each type of consumers, there are four possible types of equilibrium outcomes. Specifically, the four cases are as follows. Case 1: \((IC_h)\) and \((IC_l)\) are binding; Case 2: \((IC_l)\) and \((IR_h)\) are binding; Case 3: \((IR_l)\) and \((IC_h)\) are binding; and Case 4: \((IR_l)\) and \((IR_h)\) are binding. We discuss these four cases sequentially (We will show that only Case 3 and Case 4 can happen in equilibrium).

Recall that the IC and IR constraints are:

\[(IR_l): \rho_l(\bar{\theta}q_L - p_L - \gamma_{under}[\bar{\theta}q_H - p_H] - (\bar{\theta}q_L - p_L)] + (1 - \rho_l)(\bar{\theta}q_L - p_L) \geq 0, \]

\[(IR_h): \rho_h(\bar{\theta}q_H - p_H) + (1 - \rho_h)(\bar{\theta}q_H - p_H - \gamma_{over}[(\bar{\theta}q_L - p_L) - (\bar{\theta}q_H - p_H)]) \geq 0, \]

\[(IC_l): \rho_l(\bar{\theta}q_L - p_L - \gamma_{under}[\bar{\theta}q_H - p_H] - (\bar{\theta}q_L - p_L)] + (1 - \rho_l)(\bar{\theta}q_L - p_L) \geq \rho_l(\bar{\theta}q_H - p_H) + (1 - \rho_l)(\bar{\theta}q_H - p_H - \gamma_{over}[(\bar{\theta}q_L - p_L) - (\bar{\theta}q_H - p_H)]), \]

\[(IC_h): \rho_h(\bar{\theta}q_L - p_L - \gamma_{under}[\bar{\theta}q_H - p_H] - (\bar{\theta}q_L - p_L)] + (1 - \rho_h)(\bar{\theta}q_L - p_L) \leq \rho_h(\bar{\theta}q_H - p_H) + (1 - \rho_h)(\bar{\theta}q_H - p_H - \gamma_{over}[(\bar{\theta}q_L - p_L) - (\bar{\theta}q_H - p_H)]). \]

**Case 1:** \((IC_h)\) and \((IC_l)\) are binding.

It turns out that there does not exist \((p_h', p_l')\) which satisfy both equations simultaneously. Hence Case 1 will not happen in equilibrium.

**Case 2:** \((IR_h)\) and \((IC_l)\) are binding.

In this case, the optimal prices are

\[ p_l^* = \frac{1}{1 + \gamma_{under}\rho_l + \gamma_{over}(1 - \rho_l)} \left\{ (1 + \gamma_{over})q_H(\bar{\theta} - \theta)[\rho_h(1 - \rho_l) - \rho_l(1 - \rho_h)(1 + \gamma_{under})] + q_L(1 + \gamma_{over})\theta + \rho_l(1 + \gamma_{under})[1 + \gamma_{over}(1 - \rho_h)] - \theta - \gamma_{over}\theta[2 + \gamma_{under} - (1 + \gamma_{under})\rho_h] \right\}, \]

and
In equilibrium, the optimal prices are:

\[ p_L^* = p_L^{NR*} - \rho_l (\bar{\theta} - \theta) \gamma_{under} \frac{1 + (1 - \rho_l)q_H(1 + \rho_l \gamma_{under})}{1 + \gamma_{under} \rho_l + \gamma_{over}(1 - \rho_l)}, \]

and

\[ p_H^* = p_H^{NR*} - (\bar{\theta} - \theta) \rho_l \gamma_{under} \frac{1 + (1 - \rho_l)q_H(1 + \rho_l \gamma_{under})}{1 + \gamma_{under} \rho_l + \gamma_{over}(1 - \rho_l)}. \]

We also need to ensure that \((IR_h)\) and \((IC_l)\) are satisfied in equilibrium. One can show that \((IR_h)\) and \((IC_l)\) are true if and only if \(G(\gamma_{under}, \gamma_{over}) = \frac{1}{1 + \gamma_{under}} \left(1 + \frac{\rho_l}{1 + (1 + \gamma_{over})(1 - \rho_l)}\right) \leq \frac{q_L}{q_H}\). Therefore, Case 3 occurs in equilibrium if and only if \(G(\gamma_{under}, \gamma_{over}) \leq \frac{q_L}{q_H}\).

**Case 4.** \((IR_l)\) and \((IR_h)\) are binding.

In equilibrium, the optimal prices are:

\[ p_L^* = p_L^{NR*} - \rho_l (\bar{\theta} - \theta) \gamma_{under} \frac{1 + (1 - \rho_l)q_H(1 + \rho_l \gamma_{under})}{1 + \gamma_{under} \rho_l + \gamma_{over}(1 - \rho_l)}, \]

and

\[ p_H^* = p_H^{NR*} - (\bar{\theta} - \theta) \rho_l \gamma_{under} \frac{1 + (1 - \rho_l)q_H(1 + \rho_l \gamma_{under})}{1 + \gamma_{under} \rho_l + \gamma_{over}(1 - \rho_l)}. \]

We also need to ensure that \((IC_h)\) and \((IC_l)\) are satisfied in equilibrium. One can show that they are true if and only if \(G(\gamma_{under}, \gamma_{over}) > \frac{q_L}{q_H}\). Therefore, Case 4 occurs in equilibrium if and only if \(G(\gamma_{under}, \gamma_{over}) > \frac{q_L}{q_H}\).
Given the expressions of $p_H^*$ and $p_L^*$, it is straightforward to verify that the results in Proposition 1 are true.

**Proof of Proposition 2.**

The firm’s profit is

$$\pi^* = \alpha[p_L^* - C(q_L)] + (1 - \alpha)[p_H^* - C(q_H)]$$

$$= [\alpha p_L^* + (1 - \alpha)p_H^*] - [\alpha C(q_L) + (1 - \alpha)C(q_H)].$$

The second term, $\alpha C(q_L) + (1 - \alpha)C(q_H)$, is independent of $\gamma_{under}$ and $\gamma_{over}$, so $\frac{\partial \pi^*}{\partial \gamma_{under}} = \alpha \cdot \frac{\partial p_L^*}{\partial \gamma_{under}} + (1 - \alpha) \cdot \frac{\partial p_H^*}{\partial \gamma_{under}}$.

When $G(\gamma_{under}, \gamma_{over}) \leq \frac{q_L}{q_H}$, $\pi^* = \pi^{NR*} + \frac{(1 - \rho_h)(\bar{\theta} - \theta)(q_H - q_L)}{1 + \gamma_{under} \rho_h + \gamma_{over}(1 - \rho_h)} \cdot [(1 - \alpha)(\gamma_{under} - \gamma_{over})\rho_h - \gamma_{under}(1 + \gamma_{over})\rho_l]$. Note that $\pi^{NR*}$ is independent of $\gamma_{under}$ and $\gamma_{over}$. Therefore, $\frac{\partial \pi^*}{\partial \gamma_{under}} = \frac{\partial \left( \frac{(1 - \rho_h)(\bar{\theta} - \theta)(q_H - q_L)}{1 + \gamma_{under} \rho_h + \gamma_{over}(1 - \rho_h)} \cdot [(1 - \alpha)(\gamma_{under} - \gamma_{over})\rho_h - \gamma_{under}(1 + \gamma_{over})\rho_l] \right)}{\partial \gamma_{under}} = (1 - \rho_h)(\bar{\theta} - \theta)(q_H - q_L)$.

Similarly, $\frac{\partial \pi^*}{\partial \gamma_{over}} = -\frac{\partial \left( \frac{(1 - \rho_h)(\bar{\theta} - \theta)(q_H - q_L)}{1 + \gamma_{under} \rho_h + \gamma_{over}(1 - \rho_h)} \cdot [(1 - \alpha)(\gamma_{under} - \gamma_{over})\rho_h - \gamma_{under}(1 + \gamma_{over})\rho_l] \right)}{\partial \gamma_{over}} < 0$ always holds.

When $G(\gamma_{under}, \gamma_{over}) > \frac{q_L}{q_H}$, we know from Proposition 1 that $\frac{\partial p_H^*}{\partial \gamma_{under}} < 0$, $\frac{\partial p_H^*}{\partial \gamma_{over}} < 0$, $\frac{\partial p_L^*}{\partial \gamma_{under}} < 0$, and $\frac{\partial p_L^*}{\partial \gamma_{over}} < 0$.

Proof of Corollary 2.
We have shown that when $G(y_{under}, y_{over}) > \frac{q_L}{q_H}$, $\Delta \pi^* < 0$. When $G(y_{under}, y_{over}) \leq \frac{q_L}{q_H}$, if $y_{over} = y_{under} = \pi^{NR^*}$, because $\alpha(\gamma)$, anticipated regret are zero. Lemma 1 has shown that for given $\alpha(\gamma)$, (Lemma 2), and $\gamma$, (Under $\gamma$), $\pi^{NR^*} = (1 - \alpha)(y_{under} - y_{over})\rho_h - y_{under}(1 + y_{over})\rho_l < 0$.

Proof of Proposition 3.

Note that $G(y_{under}, y_{over}) = \frac{1}{1 + y_{under}\rho_h + y_{over}(1 - \rho_h)}$ is an increasing function of $y_{under}$ and $y_{over}$. When $G(y_{under}, y_{over}) < \frac{q_L}{q_H}$, the low-prior consumers’ expected surplus is zero, so the expected consumer surplus, $E[CS]$, is equal to the expected surplus of high-prior consumers. Therefore,

$$E[CS] = (1 - \alpha)\left\{\rho_h(\bar{\theta}q_H - p_H^*) + (1 - \rho_h)\left\{\theta q_H - p_H^* - y_{over}\left[(\theta q_H - p_H^* - (\theta q_H - p_H^*)]\right\}\right\}$$

$$= (1 - \alpha)(\rho_h - \rho_l)(\bar{\theta} - \theta)\cdot \frac{1 + y_{under}(1 + y_{over}(1 - \rho_h))q_H - (\theta q_H - p_H^*)}{1 + y_{under}\rho_h + y_{over}(1 - \rho_h)}.$$  

Take derivatives with respect to $y_{under}$ and $y_{over}$:

$$\frac{\partial E[CS]}{\partial y_{under}} = \frac{(1 - \alpha)(1 + y_{over}(1 - \rho_h))q_H - (\theta q_H - p_H^*)\rho_h - \rho_l(\bar{\theta} - \theta)}{1 + y_{under}\rho_h + y_{over}(1 - \rho_h)} < 0,$$

and

$$\frac{\partial E[CS]}{\partial y_{over}} = \frac{(1 - \alpha)(1 + y_{under}(1 - \rho_h))q_H - (\theta q_H - p_H^*)\rho_h - \rho_l(\bar{\theta} - \theta)}{1 + y_{under}\rho_h + y_{over}(1 - \rho_h)} < 0.$$

In other words, the expected consumer surplus decreases with $y_{under}$ and $y_{over}$ when $G(y_{under}, y_{over}) \leq \frac{q_L}{q_H}$.

When $G(y_{under}, y_{over}) > \frac{q_L}{q_H}$, $(IR_i)$ and $(IR_h)$ are binding, so the expected consumer surplus is a constant, zero.

Proof of Lemma 2.

Lemma 1 has shown that for given $q_H$ and $q_L$, the firm’s optimal prices when consumers do not have anticipated regret are $p_{L}^{NR^*} = [\rho_l(\bar{\theta} + (1 - \rho_l))q_L$ and $p_{H}^{NR^*} = [\rho_h(\bar{\theta} + (1 - \rho_h))q_H - (\rho_h - \rho_l)\bar{\theta}]q_L$. Plugging them into the firm’s profit function, we obtain that $\pi^{NR^*} = \alpha\left\{\rho_l(\bar{\theta} + (1 - \rho_l))q_L - C(q_L)\right\} + (1 - \alpha)\left\{\rho_h(\bar{\theta} + (1 - \rho_h))q_H - (\rho_h - \rho_l)\bar{\theta}\right\}q_L - C(q_H)\right\}$. Because $C(\cdot)$ is a convex function, $\pi^{NR^*}$ is a concave function with respect to $q_H$ and $q_L$. Therefore, the
firm’s optimal product quality levels are uniquely determined by \( \frac{\partial \pi^{\text{NR}}_h}{\partial q_L} = 0 \) and \( \frac{\partial \pi^{\text{NR}}_h}{\partial q_H} = 0 \). These conditions are equivalent to \( C'(q^{\text{NR}*}_H) = \rho_h(\bar{\theta} + (1 - \rho_h)\theta) \) and \( C'(q^{\text{NR}*}_L) = (\rho_l\theta + (1 - \rho_l)\bar{\theta}) - \frac{1 - \alpha}{\alpha} \cdot (\rho_h - \rho_l)(\bar{\theta} - \theta). \)

**Proof of Proposition 4.**

Consider the case of \( G(y_{\text{under}}, y_{\text{over}}) \leq \frac{q_L}{q_H} \). Lemma 1 shows that for given \( q_H \) and \( q_L \), the firm’s optimal prices are

\[
p^*_L = p^{\text{NR}*}_L - y_{\text{under}}(1 + y_{\text{over}})\rho_l \cdot \frac{(1 - \rho_h)(\bar{\theta} - \theta)(q_H - q_L)}{1 + y_{\text{under}}\rho_h + y_{\text{over}}(1 - \rho_h)} \quad \text{and} \quad p^*_H = p^{\text{NR}*}_H + [(y_{\text{under}} - y_{\text{over}})\rho_h - y_{\text{under}}(1 + y_{\text{over}})\rho_l] \cdot \frac{(1 - \rho_h)(\bar{\theta} - \theta)(q_H - q_L)}{1 + y_{\text{under}}\rho_h + y_{\text{over}}(1 - \rho_h)}.
\]

Plugging them into the firm’s profit function \( \pi^* = \alpha[p^*_L - C(q_L)] + (1 - \alpha)[p^*_H - C(q_H)] \), one can show that \( \pi^* \) is a concave function with respect to \( q_L \) and \( q_H \), so the firm’s optimal product quality levels are determined by \( \frac{\partial \pi^*}{\partial q_L} = 0 \) and \( \frac{\partial \pi^*}{\partial q_H} = 0 \), which are equivalent to:

\[
C'(q^{\text{NR}*}_L) = C'(q^{\text{NR}*}_L) = \frac{(1 - \rho_h)(\bar{\theta} - \theta)}{1 + y_{\text{under}}\rho_h + y_{\text{over}}(1 - \rho_h)} \cdot [(1 - \alpha)(y_{\text{under}} - y_{\text{over}})\rho_h - y_{\text{under}}(1 + y_{\text{over}})\rho_l],
\]

and \( C'(q^{\text{NR}*}_H) = C'(q^{\text{NR}*}_H) + \frac{(1 - \rho_h)(\bar{\theta} - \theta)}{1 + y_{\text{under}}\rho_h + y_{\text{over}}(1 - \rho_h)} \cdot [(1 - \alpha)(y_{\text{under}} - y_{\text{over}})\rho_h - y_{\text{under}}(1 + y_{\text{over}})\rho_l] \), where \( q^{\text{NR}*}_L \) and \( q^{\text{NR}*}_H \) are defined in Lemma 2.

Because \( C(\cdot) \) is strictly convex, \( C'(\cdot) \) is a strictly increasing function. Therefore, \( \frac{\partial q^{\text{NR}*}_L}{\partial y_{\text{under}}} < 0 \) if and only if

\[
\frac{\partial}{\partial y_{\text{under}}} \left[ \frac{(1 - \rho_h)(\bar{\theta} - \theta)}{1 + y_{\text{under}}\rho_h + y_{\text{over}}(1 - \rho_h)}([(1 - \alpha)(y_{\text{under}} - y_{\text{over}})\rho_h - y_{\text{under}}(1 + y_{\text{over}})\rho_l]\right] > 0,
\]

which is equivalent to

\[
y_{\text{over}} < \frac{(1 - \alpha)\rho_h - \rho_l}{\rho_l} \cdot \frac{1}{\rho_l}. \quad \text{Similarly,} \quad \frac{\partial q^{\text{NR}*}_H}{\partial y_{\text{under}}} > 0 \text{ if and only if } y_{\text{over}} < \frac{(1 - \alpha)\rho_h - \rho_l}{\rho_l}. \quad \text{Moreover, because}
\]

\[
\frac{\partial}{\partial y_{\text{over}}} \left[ \frac{(1 - \rho_h)(\bar{\theta} - \theta)}{1 + y_{\text{under}}\rho_h + y_{\text{over}}(1 - \rho_h)}([(1 - \alpha)(y_{\text{under}} - y_{\text{over}})\rho_h - y_{\text{under}}(1 + y_{\text{over}})\rho_l]\right] < 0 \text{ is always true when}
\]

\[
G(y_{\text{under}}, y_{\text{over}}) < \frac{q_L}{q_H}, \text{ hence } \frac{\partial q^{\text{NR}*}_L}{\partial y_{\text{over}}} < 0 \text{ and } \frac{\partial q^{\text{NR}*}_H}{\partial y_{\text{over}}} > 0 \text{ are always true.}
Following the same exercise, one can show that when $G(\gamma_{\text{under}}, \gamma_{\text{over}}) > \frac{q_L}{q_H} \frac{\partial q_L^*}{\partial \gamma_{\text{over}}} < 0$ and $\frac{\partial q_L^*}{\partial \gamma_{\text{over}}} > 0$ are always true. ■

**Proof of Proposition 5.**

We prove this proposition by providing an example where there will exist $\hat{\gamma}_{\text{over}} > 0$ and $\hat{\gamma}_{\text{under}} > 0$, such that the expected consumer surplus when $\gamma_{\text{over}} = \hat{\gamma}_{\text{over}}$ and $\gamma_{\text{under}} = \hat{\gamma}_{\text{under}}$ is higher when $\gamma_{\text{over}} = \gamma_{\text{under}} = 0$.

Specifically, let $\alpha = 0.35, \rho_h = 0.6, \rho_l = 0.2, \bar{\theta} = 1$, and $\bar{\theta} = 0.6$, and $C(q) = 0.1q^2$. When $\hat{\gamma}_{\text{under}} = 0.1$ and $\hat{\gamma}_{\text{over}} = 0.3$, the expected consumer surplus is 0.210982, while it is 0.199086 when $\hat{\gamma}_{\text{under}} = \hat{\gamma}_{\text{over}} = 0$. One can further check that the firm will always provide the full product line with these parameter values. ■

**Proof of Proposition 6.**

We prove this proposition in two separate cases, depending on the relative magnitude between $G(\gamma_{\text{under}}, \gamma_{\text{over}})$ and $\frac{q_L}{q_H}$.

**Case 1:** $G(\gamma_{\text{under}}, \gamma_{\text{over}}) \leq \frac{q_L}{q_H}$

\[
\Delta \pi_T = \rho_l(\bar{\theta} - \theta)(q_l - q_H)(1 - \rho_h) \frac{1 + \rho_l - \rho_h}{1 + \rho_l - \rho_h} - \rho_l q_L (\bar{\theta} - \theta) - \Delta \pi^*_r
\]

\[
= \rho_l(\bar{\theta} - \theta)(q_l - q_H)(1 - \rho_h) \frac{1 + \rho_l - \rho_h}{1 + \rho_l - \rho_h} - \rho_l q_L (\bar{\theta} - \theta) - (1 - \rho_h)(\bar{\theta} - \theta)(q_H - q_l) \cdot \left[ \frac{\rho_l}{1 + \rho_l - \rho_h} \cdot (\gamma_{\text{under}} - \gamma_{\text{over}}) \right] \\
= \rho_l(\bar{\theta} - \theta) \left\{ \frac{(1 - \rho_h)}{1 + \rho_l - \rho_h} - \frac{(1 - \rho_h)(\gamma_{\text{under}} - \gamma_{\text{over}})q_H - (1 - \rho_h)(\gamma_{\text{over}} - \gamma_{\text{under}})}{1 + \gamma_{\text{under}} \rho_h + \gamma_{\text{over}}(1 - \rho_h)} \right\} q_L
\]

Because $\gamma_{\text{under}} < 1$ and $\gamma_{\text{over}} < 1$, one can show that

\[
\frac{1 - \rho_h}{1 + \rho_l - \rho_h} > 0, \text{ and }
\frac{2 - 2\rho_h + \rho_l}{1 + \rho_l - \rho_h} > 0.
\]
Let $K^* = \frac{2 - 2p_H + p_L}{1 + p_L - p_H} \frac{(y_{under - over})_{p_H} - y_{under}(1 + y_{over})}{1 + p_L - p_H}$. Therefore, $\Delta \pi_{FT}^* > 0$ if and only if $q_H > K^* q_L$.

Case 2: $G(y_{under}, y_{over}) > \frac{q_L}{q_H}$

$\Delta \pi_{FT}^* = \frac{\rho_l(\bar{\theta} - \theta)(q_{H\cdot L} - q_{L})}{1 + p_L - p_H} - \rho_l q_L(\bar{\theta} - \theta) - \Delta \pi_r^*$

$= \frac{\rho_l(\bar{\theta} - \theta)(q_{H\cdot L} - q_{L})}{1 + p_L - p_H} - \rho_l q_L(\bar{\theta} - \theta) + \frac{\rho_l}{1 + y_{under} p_l + y_{over}(1 - p_H)(1 + p_L - p_H)} \cdot \{(1 - p_H)(1 + y_{over} q_{H\cdot L} - [1 - p_l + y_{over}(1 - p_H)] q_l) + \frac{\rho_l(\bar{\theta} - \theta)}{1 + y_{under} p_l + y_{over}(1 - p_H)(1 + p_L - p_H)} \cdot \{y_{over} (1 - p_H)(p_H + \rho_l y_{under}) q_{H\cdot L} - \{(1 + p_l y_{under})[p_H - p_l + y_{over}(1 - p_H)] - (1 - p_H)^2 y_{over}\} q_L\}

Note that $\frac{\rho_l}{1 + y_{under} p_l + y_{over}(1 - p_H)(1 + p_L - p_H)} > 0$ and $\frac{2 - 2p_H + p_L}{1 + p_L - p_H} + \frac{(1 - p_H) y_{under}(1 - p_H)}{1 + y_{under} p_l + y_{over}(1 - p_H)(1 + p_L - p_H)} > 0$.

Let $K^* = \frac{2 - 2p_H + p_L}{1 + p_L - p_H} \frac{(y_{under - over})_{p_H} - y_{under}(1 + y_{over})}{1 + p_L - p_H} \frac{\rho_l(\bar{\theta} - \theta)(q_{H\cdot L} - q_{L})}{1 + y_{under} p_l + y_{over}(1 - p_H)(1 + p_L - p_H)}$. Therefore, $\Delta \pi_{FT}^* > 0$ if and only if $q_H > K^* q_L$. ■

Proof of Lemma 3.

If consumer $i$ exchanges her mismatched product on the exchange market, her expected utility of buying the low-quality product (L) is: $EU_{L} = \rho_l(\overline{\theta} q_H - (p_L + p_{E_M} + e_L)) - y_{under} \max\{p_{E_M} + p_L + e_L - p_H, 0\} + (1 - \rho_l)(\theta q_L - p_L)$, and her expected utility of buying high-quality product (H) is: $EU_{H} = \rho_l(\overline{\theta} q_H - p_H) + (1 - \rho_l)(\theta q_L - (p_H - p_{E_M} + e_H)) - y_{over} \max\{p_{H} - p_L - p_{E_M} + e_H, 0\}$. If she does not exchange her product, her expected utility of buying the low-quality product (L) is: $E[U_{L}^{NE}] = \rho_l(\overline{\theta} q_L - p_L - y_{under}[(\overline{\theta} q_H - p_H) - (\overline{\theta} q_L - p_L)]) + (1 - \rho_l)(\theta q_L - p_L)$, and her expected utility of
buying the high-quality product (\(H\)) is:

\[
E[U_{iiH}^{NE}] = \rho_i(\bar{\theta}q_H - p_H) + (1 - \rho_i)(\bar{\theta}q_H - p_H - \gamma_{over} \cdot [(\bar{\theta}q_L - p_L) - (\bar{\theta}q_H - p_H)]).
\]

Given \(p_H\) and \(p_L\), the firm’s profit from the exchange market is \(\alpha(e_H + e_L)\). The firm also needs to guarantee that upgrading consumers are willing to exchange, i.e., \(\bar{\theta}q_H - (p_L + p_{EM} + e_L) - \gamma_{under} \cdot \max\{p_{EM} + p_L + e_L - p_H, 0\} \geq \bar{\theta}q_L - p_L - \gamma_{under} \cdot [(\bar{\theta}q_H - p_H) - (\bar{\theta}q_L - p_L)]\), and that the downgrading consumers are willing to exchange, i.e., \(\theta q_L - (p_H - p_{EM} - e_H) - \gamma_{over} \cdot \max\{p_H - p_L - p_{EM} + e_H, 0\} \geq \theta q_H - p_H - \gamma_{over} \cdot [(\theta q_L - p_L) - (\theta q_H - p_H)]\). Therefore, the firm’s optimal \(e_H^*\) and \(e_L^*\) must satisfy \(e_L^* = \bar{\theta}(q_H - q_L) - p_{EM}^*\) and \(e_H^* = p_{EM}^* - \bar{\theta}(q_H - q_L)\). The firm’s optimal profit from the exchange market is \(\alpha(1 - \rho_h)(\bar{\theta} - \theta)(q_H - q_L) = \frac{(1 - \rho_h)\rho_l(\bar{\theta} - \theta)(q_H - q_L)}{1 - \rho_h + \rho_l}\).