Persuasive Advertising
in a Vertically Differentiated Market

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Abstract

We study advertising content strategies in a vertically differentiated market when consumer valuation of quality is concave in quality. We consider two types of persuasive effects of the content of ads — shifting a consumer’s valuation of quality such that she values quality either more or less relative to price (valuation shifting), and shifting a consumer’s perception of quality by altering the reference point against which she evaluates quality (reference shifting). We find that a monopolist always and only prefers valuation-shifting advertising content. In a duopoly with firms selling vertically differentiated products, however, the choice is more nuanced. Specifically, the high-quality firm may not always prefer valuation-shifting content because this increases the proportion of high-valuation consumers in the market, which intensifies price competition. Furthermore, the high-quality firm may benefit from reference-shifting content, even though it reduces the perceived absolute valuation of its product, because it increases its valuation relative to the low-quality product. We also find that either the low-quality or the high-quality firm may choose not to advertise at all (under conditions that this firm would if it were a monopolist).

1 Introduction

[Advertisement] plays upon [the consumer’s] mind with studied skill, and makes him prefer the goods of one producer to those of another because they are brought to his notice in a more pleasing and forceful manner. Robinson (1969)

One of the key roles of advertisements besides conveying product information is to influence consumers’ product evaluation in a way that is more favorable to the advertising firm. Advertisements are meant to create stronger preference, and thus enhanced market power for the advertising firm. If ads indeed influence consumers, how should competing firms strategically choose their advertising messages? In this paper we study this question in the context of a vertically differentiated market.

Consider the examples in Figure [1]. An ad for Mercedes-Benz, a high-end automobile manufacturer, reads “The best or nothing” (Figure 1a). Similarly, an ad for Verizon, a telecommunications company, boasts that it has the nation’s “largest 3G network” (Figure 1c). These two ads seek to persuade buyers of cars and telecommunications services, respectively, that they should not compromise on quality when making a purchase. In contrast, an ad for a car produced by Kia, a car brand commonly associated with affordability, states that the car “was named Best Compact Car for the Money” (Figure 1b). This statement emphasizes value for money rather than quality-related features. Within the car category, while the ad by Mercedes-Benz succinctly underscores
the importance of quality, thereby inducing consumers to care less about price, the focus on “value for money” in the Kia ad creates the opposite effect where the consumers are called to trade-off quality against price. This example illustrates a typical difference in advertising content between competing firms — high-quality firms typically emphasize quality while lower-quality firms often focus on price-related features in their ads. Therefore, ads can affect consumers’ sensitivity for quality.

Another effect that ads can have is that of shifting the reference point against which quality is evaluated, i.e., the ads can influence the internal benchmark quality level that consumers invoke when evaluating quality by providing anchor points to consumers. For instance, the Verizon ad displays a map with nearly complete network coverage (and explicitly compares this with its competitor’s significantly lower network coverage), which can shift the standard against which potential phone...
service buyers evaluate network coverage. The other two ads can also be argued to influence the quality anchor albeit to a lesser extent (the Mercedes-Benz ad because it does not show or describe a car, and the Kia ad because it shows a relatively low quality car and carries a quality-price trade-off message). Past research on anchoring has demonstrated that across a wide range of domains, consumers’ evaluations of products and experiences are heavily influenced by anchor points (Epley and Gilovich 2001; Mussweiler and Strack, 2001). These anchor points have been shown to be malleable to even subtle interventions that are unrelated to the task (e.g., Ariely et al. (2003) show that subjects’ evaluations of an experience were significantly affected by the recall of their social security numbers). Furthermore, the literature on context-dependent preferences (Simonson and Tversky 1992) suggests that quality-message advertisements may shift consumers’ reference quality. Based on these ideas, we posit that quality messages in ads can shift the “quality anchor” or reference quality against which consumers evaluate quality. (We find evidence consistent with this in a lab experiment that we conducted, described later.)

The two types of advertising content effects discussed above (influencing the sensitivity to quality, or valuation, of customers and influencing the reference point for quality) are characteristic of vertically differentiated markets where firms offer products of different quality levels. Our goal in this paper is to understand the strategic impact of these different ad effects in a competitive setting. Also, while a particular ad can be expected have both effects simultaneously, ads can be designed that have more of one effect than the other, and we investigate and provide guidance on how firms should, under different market conditions, optimally choose ad content corresponding to each type of effect. Overall, our work falls under the umbrella of persuasive advertising in the context of vertically differentiated markets.

We develop a game theory model in which two firms endowed with different levels of product quality decide their ad strategies and compete on price. Consumers are heterogeneous in their valuations for quality, have marginally reducing valuation for quality, and evaluate quality relative to a reference point. Ad content can have two types of effects—shifting a consumer’s valuation of quality such that she values quality either more or less relative to price (valuation shifting), and shifting a consumer’s perception of quality by altering the reference point against which she evaluates quality (reference shifting). Our analysis shows that a monopolist always chooses ad content that increases
consumer valuation of quality. This strategy allows the monopolist to extract greater surplus from the consumers by charging them higher prices post-ad. The monopolist never chooses ad content that increases the reference quality because doing so would only reduce the consumers’ perceived value of quality.

However, the insights are quite different in a competitive setting. We show that under certain conditions, competing firms may choose valuation-shifting ad content that decreases consumers’ valuation of quality—while this reduces the extractable surplus, it may enhance differentiation between firms. Furthermore, while the monopolist’s profit strictly decreases with the reference quality, firms in competition may benefit from a higher reference quality even though it undermines consumers’ perceived value of quality. In particular, we find that under certain conditions, the high-quality firm in a duopoly chooses reference-shifting as content to raise the reference quality, which again may lead to increased product differentiation.

Finally, a key result that emerges from our analysis concerns the relationship between consumer surplus and competition. Standard economic theory suggests that as competition intensifies, consumer surplus increases as product prices fall. This reasoning, however, rests on the premise that consumer valuation of the product remains constant. In the context of persuasive advertising, firms’ advertising strategies change consumers’ perceptions and product valuations. We show that even if competition intensifies and prices fall, a switch in advertising regimes may cause consumers to value quality less such that the overall perceived consumer surplus decreases.

Our work is related to the literature on persuasive advertising under competition. In this stream, a large body of research has studied the “combative” role of advertising where horizontally differentiated firms decide the level of spending on ads that shift consumer preferences (Bloch and Manceau, 1999; Tremblay and Martins-Filho, 2001; Tremblay and Polasky, 2002; Chen et al., 2009). Surprisingly, little attention has been paid to the persuasive role of advertising in a vertically differentiated market (Bagwell, 2005); we contribute to this by considering two types of persuasive effects of ads: valuation shifting and reference shifting.

We assume diminishing marginal utility of quality for consumers. This assumption is supported by Kahneman and Tversky (1979), who argue that Prospect Theory, which features a concave value
function in the gains domain, is applicable not only to monetary outcomes, but also “to choices involving other attributes; e.g., quality.” Furthermore, to the extent that quality can be viewed as a form of stimulus experienced by the consumer (i.e., the perceived level of happiness the consumer feels for a given level of quality), the diminishing marginal return to quality is supported by the psychology literature, which suggests that the marginal change in hedonic response to a stimuli “decreases with the distance from the reference point” (Kahneman and Tversky, 1979; Tversky and Kahneman, 1991; Frederick and Loewenstein, 1999). Our results demonstrate that the concavity of the consumer utility function yields important insights concerning advertising strategies that do not emerge when a linear utility function is used, which has been typically assumed in previous work on vertical differentiation (Mussa and Rosen, 1978; Shaked and Sutton, 1982; Moorthy, 1988; Vandenbosch and Weinberg, 1995; Wauthy, 1996).

We allow competing firms’ marketing actions, specifically ads, to shift consumers’ reference quality and determine how firms should optimally use ads to influence reference quality. This is related to Kopalle and Lehmann (1995), Kopalle and Assunção (2000) and Kopalle and Lehmann (2006), who study the impact of setting reference quality on firm strategy in a monopoly setting. Work on reference-dependent quality evaluations is also related to our research; specifically, Hardie et al (1993) and Kopalle and Winer (1996) use the concept of reference-dependent quality evaluations in the contexts of choice models and dynamic product quality decisions, respectively. We note that this is distinct from research on the effects of reference prices on firm behavior (Hardie et al., 1993; Kopalle and Winer, 1996; Kopalle et al., 1996; Ho et al., 2006; DellaVigna and Gentzkow, 2009; Baucells et al., 2011).

A large literature studies the informative role of advertising through signaling when consumers are not perfectly informed about the product (Nelson, 1974; Kihlstrom and Riordan, 1984; Milgrom and Roberts, 1986; Mayzlin and Shin, 2011). In our framework, consumers know absolute quality and there is no asymmetric information. Previous work that has examined the content of advertising also falls under the umbrella of the informative role of advertising. For example, Anderson and Renault (2006) study how much product information firms should disclose to consumers in their ads, and show that firms should not fully reveal match information to consumers because doing so creates a hold-up problem and results in a reduction of store visits.
The rest of the paper is organized as follows. Section 2 develops the basic model used in our analysis. Section 3 examines the benchmark model of monopoly. The benchmark analysis will allow us to disentangle the impact of competition on the firms’ decisions. Section 4 investigates the main model of competition where firms choose ad types and set prices. Section 5 assesses the robustness of the main results by examining various extensions. Finally, Section 6 discusses managerial implications and concludes with suggestions for future research. For ease of exposition, we relegate all proofs and lengthy expressions to the appendix.

2 Model

Firms

The game consists of two firms, each producing a good of distinct quality level, and two consumer segments with different valuations of quality. To focus on the role of advertising, we assume that the quality of the firms’ products is exogenously determined. We denote the qualities of the higher- and lower-quality products by $q_H$ and $q_L$, respectively, where $q_H > q_L > 0$. We refer to the firm that produces the higher quality product as Firm $H$, and its lower quality counterpart as Firm $L$. The marginal cost of producing $q_H$ is $c > 0$ and that of producing $q_L$ is normalized to zero. Each firm decides whether to show an ad or not, denoted by $a_H$ and $a_L$ for Firm $H$ and Firm $L$, respectively, where $a_H$ and $a_L$ are binary and 1 denotes showing an ad and 0 denotes not showing an ad. If a firm advertises, it also chooses its ad content. Finally, firm $H$ and $L$ set prices $p_H$ and $p_L$ in competition. We discuss all decisions and their timings in detail as we develop the model.

Consumers

We assume that for a consumer the utility of consuming a product of quality $q$ is given by

$$u_t(q) = \theta_t v(q - q_r),$$

where $t$ is the type of the consumer, $\theta_t$ is a valuation multiplier determined by the type, $q_r$ is the reference quality ($0 < q_r \leq q$), and $v(\cdot)$ is an increasing function with $v(0) = 0$ and $v''(\cdot) < 0$.\footnote{See Mussa and Rosen (1978) for a discussion on similar forms of generalized utility functions for quality.}
that the assumptions on \( v(\cdot) \) imply that consumers have diminishing marginal utility of quality.

Regarding consumer types, we assume that a unit mass of consumers is distributed such that \( \alpha \) proportion have a high valuation multiplier \( \theta_h \), and the other \( 1 - \alpha \) proportion have low valuation multiplier \( \theta_l \), which we normalize to 1; i.e., \( \theta_h > 1, \theta_l = 1 \) and \( \alpha \in (0,1) \). While the distribution parameter \( \alpha \) is common knowledge, only the consumers know their own valuation, \( \theta \in \{ \theta_h, \theta_l \} \).

We index consumer type by \( t \in \{ h, l \} \), which corresponds to consumers with valuations \( \theta_h \) and \( \theta_l \), respectively. Finally, there exists an outside option that yields utility \( u_0 \), which we normalize to zero; this would correspond to, say, not making any purchase. We assume that the firms’ ads do not influence the utility of the outside option. Figure 2a illustrates the utility curves for the two consumer types. If a consumer purchases the product from firm \( T \in \{ H, L \} \), she pays price \( p_T \) which leads to a disutility of \( p_T \) (i.e., utility of \( -p_T \)). Consumers compare the net utilities from the products with qualities \( q_H \) and \( q_L \) and from the outside option, and choose whichever yields the highest net utility.

**Types of Advertising**

As discussed in the introduction, we consider two effects of ads — valuation shifting and reference shifting — and while an ad can have both types of effects, a particular ad can have content to emphasize one type of effect more than the other. Based on this argument, we make a stylized assumption for our main model that an ad can have purely one type of effect; therefore, for our main model, we consider two qualitatively distinct types of persuasive ads: valuation-shifting ads and reference-shifting ads. Making this assumption enables us to communicate the key insights cleanly, and in later analysis we show that allowing ads to have both effects simultaneously does
Next, we discuss how we operationalize valuation shifting and reference shifting. A *valuation-shifting ad* redistributes the proportion of high- and low-valuation consumers in the population by changing $\alpha$. Depending on the firm’s ad content, the proportion of high-valuation consumers, $\alpha$, could either increase or decrease. For example, ads that emphasize the importance of quality (e.g., the Mercedes-Benz ad in Figure 1a) increase $\alpha$ by inducing larger segments of the population to become more concerned about absolute quality. We denote this type of valuation-shifting ad by $V^h$ and assume that it increases the proportion of high-valuation consumers by a magnitude $\gamma^h$. On the other hand, ads that highlight value (e.g., the Kia ad in Figure 1b) have the effect of making consumers less concerned about absolute quality and more concerned about the price/quality trade-off. We denote this type of valuation-shifting ad by $V^l$ and assume that it increases the proportion of low-valuation consumers by a magnitude $\gamma^l$. We assume that $\gamma^h = \gamma^l = \gamma$; this assumption is for analytical simplicity and does not alter the key trade-offs captured and insights obtained by the model. If one firm chooses $V^h$ and the other $V^l$, the advertising effects negate one another and the distribution of consumer valuation remains unchanged.

The second type of ad is a *reference-shifting ad*, which we denote by $R$. This form of advertising alters consumers’ perception of quality by changing the reference point against which consumers evaluate quality (e.g., the Verizon ad in Figure 1c), i.e., it influences the internal benchmark quality level that consumers invoke when evaluating quality. As discussed earlier, this is motivated by the streams of behavioral research on anchoring effects ([Epley and Gilovich 2001] [Mussweiler and Strack 2001] [Ariely et al. 2003] and context-dependent preferences ([Simonson and Tversky 1992]).

Notably, reference point formation has been conceptualized as the centroid of the relevant characteristics of all products considered ([Bodner and Prelec 2001]) or a point in the “convex hull of the existing alternatives” ([Orhun 2009]). We invoke such a characterization, and model the consumers’ reference quality as

$$q_r(a_H, a_L | a_0) = \frac{a_0 \delta_0 + a_L \delta_L + a_H \delta_H}{a_0 + a_L + a_H},$$

where $\delta_L$ and $\delta_H$ denote the quality anchor points communicated by the ads of Firms $L$ and $H$, respectively.
respectively, and $a_L$ and $a_H$ denote the advertising intensities of Firms $L$ and $H$, respectively. The parameter $a_0$ represents the weight of a pre-existing reference quality level $\delta_0$. To illustrate, if both firms advertise, then $a_L = a_H = 1$ and the resultant reference quality is $\frac{a_0 \delta_0 + \delta_L + \delta_H}{a_0 + 2}$; if neither firm advertises, then $a_L = a_H = 0$ and the reference quality does not change and remains at $\delta_0$.

Before we proceed further, we elaborate on the implications of reference-shifting ads. As illustrated in Figure 2b, reference-shifting ads increase the reference quality (so long as the pre-existing reference quality level, $\delta_0$, is sufficiently low), which in turn lowers consumers’ perceived utility for quality. One might ask, could exposure to an ad about quality, in practice, lower consumers’ utility for quality? If so, would firms ever design and show ads that have such a seemingly negative effect?

First, consider the question: Could exposure to an ad lower consumers’ utility for quality? Our argument that ads can shift the quality reference point imply that if a consumer were shown an ad featuring a low-quality product versus an ad showing a high-quality product, the quality reference point would shift more in the latter case, leading to the consumer perceiving a lower WTP for the same product in the latter case compared to the former case. To see if this can happen, we conduct a between-subjects experiment with Amazon MTurks in which subjects were exposed to exactly one ad of a robotic vacuum cleaner while being made to read an unrelated article, and were then asked for their WTP for another robotic vacuum cleaner. In Condition 1, a low-quality vacuum cleaner was featured in the ad, while in Condition 2, a high-quality vacuum cleaner was featured in the ad (in both conditions, the ad contained a few keywords describing the product but contained no price information). In both conditions, subjects were subsequently asked to specify their WTP for the low-quality vacuum cleaner on a scale ranging from $0$ to $1,000$. (See Appendix A4 for more details on the experiment.) According to our arguments, the average WTP should be lower in Condition 2. Consistent with this, we find that the average WTP in Condition 2 is $31.7\%$ lower compared to Condition 1 ($t = 2.27, p < 0.03$; Conditions 1 and 2 had 31 and 37 subjects, respectively). While this is not a thorough investigation (which is out of the scope of this paper), the results provide

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2Our specification of reference quality in (2) is consistent with the reference point formation process described in the reference price literature (Kalyanaram and Winer, 1995; Kopalle and Winer, 1996; Mazumdar et al., 2005). In particular, the literature highlights two critical components of reference prices: (i) past prices observed by consumers and (ii) “the recency effect of prior exposures to price.” That is, the more recent the observed price, the more prominently that price is encoded in the reference price. Analogously, we can interpret the $\delta$s as previously observed quality levels which are encoded more saliently in the reference quality with advertisement exposures. Note, however, that we do not consider reference price effects as our aim is to focus on quality considerations.
preliminary evidence in line with the theory that exposure to ads may, indeed, reduce consumers’ utility for a particular quality level. We believe, in fact, that this is a remarkable result since we only show one ad, which is a very weak treatment, and we still obtain the posited effect.

Second, consider the question: Why would a firm have an incentive to show an ad that lowers consumers’ utility for quality? As we will demonstrate in the next section, a monopolist would never choose a reference-shifting ad, as it only reduces the extractable consumer surplus. However, in a duopoly in which vertically differentiated firms compete, reference-shifting ads may emerge in equilibrium. Looking ahead a little bit, the intuition is that even though a reference-shifting ad decreases consumers’ absolute perceived utility for quality, showing this ad may place a firm in a better competitive position and thereby help improve its profit. The strategic motivation behind employing a reference-shifting ad is comparable to that of predation strategies incumbents deploy in order to reduce competition. For example, firms may lower their prices below marginal cost (Milgrom and Roberts, 1982) or “overinvest” in production technology (Fudenberg and Tirole, 1984) in order to thwart entry. Employing a reference-shifting ad can be viewed as one such strategy, which will never be seen in a monopoly situation, and if and when seen in a duopoly situation is actually helping the firm showing the ad by placing it in a better competitive position.

Going back to the model specification, we now make some simplifications by assuming that $\delta_L = \delta_H = \delta$ and $\delta_0 = a_0 = 0$ (these assumptions are for ease of exposition can be relaxed without changing the qualitative insights of the paper). Under this simplification, if either or both firms choose reference-shifting advertising, the resultant reference quality shifts to $q_r = \delta \left( = \frac{\delta}{1 + \delta} \right)$, otherwise it remains at $q_r = 0$. We assume that the highest reference quality, $\delta$, must be lower than $q_L$, i.e., we impose the condition $\delta < q_L$.

We can now combine the firm’s choice of whether to advertise or not, if yes, which type of ad to show, into one action, which is to choose the type of ad $a \in \{V^h, V^l, R, \emptyset\}$, where $\emptyset$ means no advertising. We assume that if the firm advertises, it incurs a cost $k > 0$, which is the same for any type of ad.

We note that in Section 5 we relax two assumptions about ad-related actions—in Section 5.1
we allow a single ad to contain both valuation-shifting and reference-shifting elements, and in
Section 5.2, we allow a firm to implement both types of ads by allocating its budget across different
ads. The results that we obtain are similar to those in the main model.

### Game Timing

Firms play a two-stage game. In the first stage, firms simultaneously decide ad types $a \in \{V^h, V^l, R, \emptyset\}$. In the second stage, firms observe the decisions from the first stage and simultaneously set prices. After this, profits are realized and the game ends. We use subgame-perfect Nash equilibrium (SPNE) as the solution concept.

### 3 Benchmark: Monopoly

We begin our analysis with the monopoly case, which will serve as a useful benchmark to help understand more nuanced results under duopoly. Consider a monopolist that produces quality $q$ at marginal cost normalized to 0. The monopolist decides the type of ad $a \in \{V^h, V^l, R, \emptyset\}$ and then sets price $p$.

In the pricing stage, the monopolist has two strategies at hand: (i) charge a low price and serve the whole market, or (ii) charge a high price and extract high surplus from $h$-type consumers at the expense of foregoing demand from the $l$-type consumers.

For ease of exposition, we simplify notations for post-advertising variables as follows. The post-advertising proportion of high-valuation consumers and the post-advertising reference quality, respectively, are

$$\alpha^a = \begin{cases} 
\alpha + \gamma & \text{if } a = V^h, \\
\alpha - \gamma & \text{if } a = V^l, \\
\alpha & \text{otherwise}, 
\end{cases} \quad q^a_r = \begin{cases} 
\delta & \text{if } a = R, \\
0 & \text{otherwise}. 
\end{cases} \quad (3)$$
The monopolist’s optimal price and the associated profit, respectively, are

\[ p^*(a) = \begin{cases} v & \text{if } v(q - q^a_1) > \alpha^a \theta_h v(q - q^a_2), \\ \theta_h v & \text{if } v(q - q^a_1) \leq \alpha^a \theta_h v(q - q^a_2), \end{cases} \]

\[ \pi^*(a) = \max \left[ v(q - q^a_1), \alpha^a \theta_h v(q - q^a_2) \right] - k_{\{a \neq \emptyset\}}. \] (4)

Given the second stage outcome, the monopolist decides in the first stage between valuation-shifting advertising, reference-shifting advertising, and no advertising. The following proposition states the monopolist’s strategy.

**Proposition 1 (Monopolist Advertising).** If the monopolist advertises, it always chooses valuation-shifting advertising that increases the proportion of high-valuation consumers.

The intuition behind the monopolist’s strategy is that choosing \( V^h \) increases the proportion of consumers with high valuation, which means that the monopolist can extract a larger surplus in the subsequent pricing stage. \( V^l \) has the opposite effect of reducing the proportion of consumers with larger extractable surplus, and is thus dominated by \( V^h \). On the other hand, increasing the reference quality via \( R \) only serves to dampen consumer willingness-to-pay for quality as consumers evaluate quality against a higher standard, as illustrated in Figure 3. This effect, which we refer to as the **devaluation effect**, exerts a downward force on price and hence lowers the monopolist’s profit.

In the next section, we show that in contrast to the monopoly case, when competition is introduced, firms may raise the reference quality via reference-shifting advertising. Furthermore, competitive
forces may undermine Firm $H$’s incentive to increase the proportion of $h$-type consumers, despite the fact that Firm $H$ serves the $h$-type consumers in equilibrium.

4 **Competition: Duopoly**

In the duopoly scenario, Firm $H$ produces the product with high quality, $q_H$, and Firm $L$ produces the product with low quality, $q_L$, where $q_H > q_L > 0$\footnote{We restrict our attention to the interesting part of the parameter space for which: (i) Firm $H$ does not always serve the whole market, and (ii) the marginal cost of producing high quality product is sufficiently low to rule out the case where $V^h$ is dominated for Firm $H$. The expressions and derivations of these conditions are provided in Section AI in the appendix.} We denote the post-advertisement proportion of $h$-type consumers and reference quality, respectively, as

$$
\alpha^{a_H,a_L} = \begin{cases} 
\alpha + 2\gamma & \text{if } a_H = a_L = V^h, \\
\alpha + \gamma & \text{if } a_T = V^h, a_{-T} \in \{R, \emptyset\}, \\
\alpha & \text{if } a_T = V^l, a_{-T} = V^l, \\
& \text{or } a_H, a_L \in \{R, \emptyset\}, \\
\alpha - \gamma & \text{if } a_T = V^l, a_{-T} \in \{R, \emptyset\}, \\
\alpha - 2\gamma & \text{if } a_H = a_L = V^l,
\end{cases}
$$

and

$$
q^{a_H,a_L}_r = \begin{cases} 
0 & \text{if } a_H \neq R \neq a_L, \\
\delta & \text{otherwise},
\end{cases}
$$

where $a_H$ and $a_L$ represent the advertisement types chosen by Firms $H$ and $L$, respectively. For ease of exposition, we suppress the first-stage variables $(a_H, a_L)$ when discussing second-stage results. In particular, we let $v_T \triangleq v(q_T - q^{a_H,a_L}_r)$ denote the post-advertisement valuation of quality $q_T$, and $\alpha \triangleq \alpha^{a_H,a_L}$ the post-advertisement proportion of high-valuation consumers.

4.1 **Pricing Subgame**

We begin our analysis from the pricing stage game and proceed backwards to solve for the SPNE. To help develop intuition for the equilibrium pricing strategy, it is useful to examine how prices affect consumer demand. As illustrated in Figure 4, demand outcomes can be partitioned into five regions. For instance, if Firm $H$’s price is high and Firm $L$’s low, then, all else equal, both the $h$-
and l-type consumers purchase the cheaper low-quality product from Firm L (Region I). Conversely, if Firm H’s price is low and Firm L’s high, then both types of consumers buy from Firm H (Region V). And for price ranges in between the two regions, the two consumer segments separate such that each consumer segment chooses different alternatives. In particular, for intermediate ranges of $p_H$ and low $p_L$, the high-valuation consumers purchase the high-quality product and the low-valuation consumers purchase the low-quality product. The expressions for the boundaries delineating the five regions are provided in Section A2 in the appendix.

Given the demand for each pair of prices, firms simultaneously set prices. First, note that there is no pure strategy equilibrium in the pricing subgame. This is due to the discrete characterization of consumer types which leads to discontinuous changes in demand when firms undercut each other’s price (e.g., Narasimhan 1988, Raju et al. 1990). However, there exists a unique mixed strategy equilibrium such that firms choose prices within a certain range according to some distribution function. The following lemma characterizes the pricing subgame equilibrium.

**Lemma 1 (Price Subgame Equilibrium).** There does not exist a pure strategy equilibrium. There exists a unique mixed strategy equilibrium characterized by the following cumulative distribution

\[
\theta(l(v_H - v_L)) = \theta(h(v_H - v_L)) = \theta(hv_H) = \theta(lv_H) = \theta(p_L) = \theta(p_H) = \theta(V)
\]
functions for Firms H and L, respectively:

\[
F^*_H(p) = \begin{cases} 
0 & \text{if } p < p_H, \\
\frac{1}{\alpha} - \frac{(1-\alpha)v_L}{\alpha(p-H)} & \text{if } p_H \leq p < \bar{p}_H, \\
1 & \text{if } \bar{p}_H \leq p,
\end{cases}
\quad \text{and } F^*_L(p) = \begin{cases} 
0 & \text{if } p < p_L, \\
\frac{p-(1-\alpha)v_L}{p_H+(1-\alpha)v_L} & \text{if } p_L \leq p < \bar{p}_L, \\
1 & \text{if } \bar{p}_L \leq p,
\end{cases}
\]

where \( p_H = \theta_h(v_H - v_L) + (1-\alpha)v_L \), \( \bar{p}_H = \theta_h(v_H - v_L) + v_L \), \( p_L = (1-\alpha)v_L \), and \( \bar{p}_L = v_L \).

The equilibrium price distributions \( F^*_H(p) \) and \( F^*_L(p) \) are plotted in Figure 5 for two different levels of \( \alpha \). Firm H uses prices within the interval \([\theta_h(v_H - v_L) + (1-\alpha)v_L, \theta_h(v_H - v_L) + v_L]\), and Firm L within \([(1-\alpha)v_L, v_L]\) with a mass point equal to \( \frac{\theta_h(v_H - v_L) + (1-\alpha)v_L - c}{\theta_h(v_H - v_L) + v_L - c} \) at \( v_L \). An interesting feature of these price distributions is how they vary with respect to changes in \( \alpha \). Specifically, as \( \alpha \) increases, both firms’ price distributions decrease in first-order stochastic dominance (see Figure 5). In other words, firms assign more probability weights to lower prices as the proportion of high-valuation consumers increases. We state this result in the following lemma.

**Lemma 2.** As the proportion of high-valuation consumers increases, both firms’ price distributions decrease in first-order stochastic dominance.

One may conjecture that as a larger proportion of consumers value quality more, firms will set higher prices to extract the additional consumer surplus. Intriguingly, the proposition suggests that prices decrease when there are more high-valuation consumers. The intuition is that a larger \( \alpha \) implies a smaller \( l \)-type consumer base for Firm L to serve. Therefore, Firm L increasingly targets the high-valuation consumers with low prices. Firm H rationally anticipates Firm L’s pricing strategy and, in an effort to retain its \( h \)-type consumers, lowers its own price correspondingly.
In essence, an increased concentration of consumers toward the high-valuation segment intensifies price competition.

Lemma 2 suggests an interesting implication for Firm $H$’s advertising strategy. Namely, Firm $H$ will not be unequivocally better off choosing valuation-shifting advertising to alter consumers’ tastes toward higher quality, despite the fact that Firm $H$ serves the $h$-type consumers. Indeed, we will later derive conditions under which Firm $H$ does not engage in valuation-shifting advertising.

Given the equilibrium price distributions above, it can be shown that the subgame equilibrium profits of Firms $H$ and $L$, respectively, are

$$
\mathbb{E}[\pi_H] = \alpha (\theta_h (v_H - v_L) + (1 - \alpha)v_L - c) - k\mathbb{I}_{\{a_H \neq \emptyset\}}, \quad (6)
$$

$$
\mathbb{E}[\pi_L] = (1 - \alpha)v_L - k\mathbb{I}_{\{a_L \neq \emptyset\}}. \quad (7)
$$

With the subgame pricing strategies at hand, we are ready to analyze the firms’ first-stage advertising decisions, which we turn to next.

### 4.2 Advertising Subgames

Recall that each firm chooses from a set of four different advertising strategies, $\{V^h, V^l, R, \emptyset\}$, denoting quality ads that increase the proportion of high-valuation consumers, increase the proportion of low-valuation consumers, increase the reference quality, and no advertising, respectively. This means that there are $4 \times 4 = 16$ subgames to analyze for the advertising stage. In this section, we first show that the number of equilibrium candidate subgames can be reduced to 6 by characterizing the dominated strategies of each firm. Secondly, we present two additional intermediary results that are instrumental in developing intuition for the firms’ endogenous advertisement choices in equilibrium (that are subsequently presented in Section 4.3).

#### Dominated and Dominant Ad Strategies

**Lemma 3.** Firm $H$ never chooses $V^l$. 

Increasing the proportion of low-valuation consumers is dominated for Firm $H$. Intuitively, by increasing the proportion of low-valuation consumers, larger demand would be drawn to Firm $L$ that offers lower prices. Firm $H$ is better off advertising to increase the high-valuation consumer segment. Thus, we can eliminate $V^l$ from Firm $H$’s strategy set.

**Lemma 4.** If Firm $L$ advertises, it only chooses $V^l$.

Lemma 4 implies that $V^h$ and $R$ are ruled out from Firm $L$’s strategy set. In contrast to Firm $H$’s strategy, increasing the proportion of high-valuation consumers adversely affects Firm $L$’s profit. This is because doing so would make a larger proportion of the consumers find the higher quality product more attractive. Moreover, increasing reference quality via reference-shifting advertising hurts Firm $L$’s profit because higher reference point decreases consumers’ willingness-to-pay for quality. On the other hand, increasing the proportion of low-valuation consumers improves Firm $L$’s profit as Firm $L$ can target a larger consumer base with its low price. Therefore, provided the advertising cost is not too high, Firm $L$ will choose $V^l$.

Combining the results from Lemmas 3 and 4, we obtain the reduced strategy sets $S_H = \{V^h, R, \emptyset\}$ and $S_L = \{V^l, \emptyset\}$ for Firms $H$ and $L$, respectively. In sum: (i) only Firm $H$ considers reference-shifting advertising, (ii) Firm $H$’s valuation-shifting advertising increases $\alpha$, and (iii) Firm $L$’s valuation-shifting advertising decreases $\alpha$.

**Profitability of Valuation- and Reference-Shifting Ads**

To develop an understanding of the firms’ incentives to choose each type of advertising, we analyze how firm profits change with respect to the proportion of high-valuation consumers and the reference quality, holding other parameters fixed. Again, for ease of exposition, we let $v_T \triangleq v(q_T - q_r^{H,L})$ denote the post-advertisement valuation of quality $q_T$ in contexts where the reference quality, $q_r$, need not be made explicit.

**Lemma 5.** Firm $L$’s profit increases monotonically with the proportion of low-valuation consumers. On the other hand, Firm $H$’s profit increases with the proportion of high-valuation consumers if and only if $\alpha < \frac{\theta_h((v_H - v_L) + v_L - c)}{2v_L}$.
This lemma offers interesting insights. While Firm $L$’s profit monotonically increases in the proportion of the $l$-type consumers, Firm $H$’s profit does not increase monotonically in the proportion of the $h$-type consumers. To understand this, consider the two distinct effects of increasing $\alpha$ on Firm $H$’s profit. First, since Firm $H$ serves the $h$-type consumers in equilibrium, higher $\alpha$ translates to higher demand; therefore, increasing $\alpha$ has a positive effect on Firm $H$’s profit. On the other hand, increasing $\alpha$ also has an indirect, negative effect on Firm $H$’s margin, $\theta_h(v_H-v_L)+(1-\alpha)v_L-c$ (from profit expression (6)). Specifically, increasing $\alpha$ reduces the second component of Firm $H$’s margin, $(1-\alpha)v_L$, which is the minimum price Firm $L$ charges in its mixed strategy. The price decrease is attributable to the pro-competitive effect of increasing the concentration of high-valuation consumers as discussed in Lemma 2. The interplay of these two countervailing forces leads to the concavity of Firm $H$’s profit with respect to $\alpha$, as illustrated in Figure 6.

Therefore, while Firm $L$ unequivocally benefits from increasing the proportion of $l$-type consumers, Firm $H$ exercises more caution in trading off the countervailing effects of increasing the proportion of $h$-type consumers. Lemma 5 suggests that Firm $H$ will consider valuation-shifting advertising only if $\alpha$ is not too large. If the proportion of $h$-type consumers is large, then further increasing $\alpha$ via valuation-shifting advertising will exacerbate the competition effect to the extent that, even though Firm $H$’s demand increases, reduced margins ultimately leads to lower profit.

Next, we examine how firm profits change with respect to reference quality, $q_r$. The following lemma demonstrates that, in contrast to the monopoly case, increasing the reference quality does not always reduce firm profits in a duopoly. Under certain conditions, Firm $H$ benefits from a higher reference quality even though it reduces consumers’ utility for quality.

**Lemma 6.** Firm $L$’s profit decreases monotonically with the reference quality. On the other hand,
Firm H’s profit increases with the reference quality if and only if

\[ \theta_h (v'(q_L - q_r) - v'(q_H - q_r)) - (1 - \alpha)v'(q_L - q_r) > 0. \]  

(8)

To understand this result, consider the first term on the left-hand side of (8), \( \theta_h (v'(q_L - q_r) - v'(q_H - q_r)) \), which is derived from differentiating \( \theta_h (v(q_H - q_r) - v(q_L - q_r)) \) with respect to \( q_r \). From Firm H’s perspective, this term represents the reference-shift-induced change in the premium that Firm H charges to \( h \)-type consumers. Alternatively, it is the change in the additional utility that \( h \)-type consumers receive from consuming \( q_H \) instead of \( q_L \). Insofar as consumer marginal utility of quality diminishes (i.e., \( v''(\cdot) < 0 \)), this change in premium is always positive. We will hereafter refer to this positive change in Firm H’s premium as the premium effect. The premium effect forms the basis for Firm H’s incentive to increase reference quality.

Figure 7 illustrates how an increase in reference quality can widen the difference in quality valuations: shifting the reference quality by \( \delta \) increases the perceived quality differential from \( \Delta \) to \( \Delta' \). And enlarging the perceived difference in qualities allows Firm H to charge higher premiums to the \( h \)-type consumers, thereby increasing Firm H’s profit.

Increasing the reference quality, however, is not without a cost for Firm H. The negative effect of a higher reference quality is captured by the second term on the left-hand side of (8): \( (1 - \alpha)v'(q_L - q_r) \). This term represents the rate of decline in Firm L’s price (or more precisely, the lowest price Firm L charges in the mixed strategy) in response to the reference-shift-induced devaluation of its product. With Firm L’s price lower, Firm H is now forced to reduce its own price as well in order to prevent \( h \)-type consumers from switching to Firm L. We hereafter refer to this reference-shift-
induced reduction in Firm H’s margin as the devaluation effect. In effect, the trade-off between the premium effect and the devaluation effect will determine Firm H’s decision to choose reference-shifting advertising.

Note that the premium effect described in Lemma 6 depends crucially on the property of diminishing marginal utility of quality. For example, if \( v(\cdot) \) were linear such that the marginal utility of quality is constant, then increasing the reference quality would always reduce Firm H’s profit. The intuition is that when the value function is linear, the valuations of both low and high quality products decrease by the same magnitude. The premium effect, whose value derives from the asymmetric decline in quality valuations, would then disappear and only the negative devaluation effect would remain. In this case, reference-shifting would always decrease both firms’ profits.

This feature of our model represents an important departure from previous models of vertical differentiation in the literature. To the best of our knowledge, all studies that address vertical differentiation assume a linear valuation of quality (e.g., Mussa and Rosen 1978; Shaked and Sutton 1982; Moorthy 1988; Vandenbosch and Weinberg 1995; Wauthy 1996). Our result shows that incorporating diminishing marginal utility into consumer valuation of quality may generate novel managerial insights related to firm strategies.

Next, we examine how the profitability of the firms’ advertising strategies varies with respect to the key exogenous parameters in the model. The following proposition summarizes the comparative statics results.

**Proposition 2.**

- The profitability of Firm H’s valuation-shifting advertising (i) decreases with \( q_L \), (ii) increases with \( q_H \), (iii) increases with \( \theta_h \), and (iv) decreases with \( \alpha \).

- The profitability of Firm H’s reference-shifting advertising (i) decreases with \( q_L \), (ii) increases with \( q_H \), (iii) increases with \( \theta_h \), and (iv) increases with \( \alpha \) for sufficiently large premium effect.

- The profitability of Firm L’s valuation-shifting advertising increases with \( q_L \).

---

5 More formally, linearity implies that the left-hand side of (8) would collapse to \( -\theta_h(1 - \alpha)v'(q_L - q_v) \), and since this value is always negative, condition (8) would never be satisfied.
The comparative statics for the profitability of valuation-shifting advertising for both firms is fairly straightforward. The profitabilities are consistent with the direction of improvement in the advertising firm’s margin. For example, since higher $q_H (q_L)$ implies higher margins for Firm $H$ (Firm $L$), increasing the advertising firm’s demand base via valuation-shifting advertising is commensurately more profitable.

The results are slightly more nuanced for Firm $H$’s reference-shifting advertising. Why does the profitability of its reference-shifting advertising decrease with $q_L$? Recall that the profitability of reference-shifting ads hinges on the *premium effect*, which emerges from the asymmetric decline in quality valuations as the reference quality increases. If $q_L$ is high such that it is close to $q_H$, then an increase in reference quality would reduce the valuations for both qualities by similar magnitudes. Consequently, the perceived difference between two qualities would only be marginally increased as a result of the reference-shifting advertising, thus rendering the *premium effect* negligible.

On the other hand, if $q_L$ is low, then $q_L$ would be subject to a steeper devaluation due to the property of diminishing marginal utility, whereas $q_H$, farther out in the flatter region of the utility curve, would be relatively immune to such devaluation. This asymmetric devaluation would lead to significantly enlarged difference in perceived quality valuations between the two products. Therefore, the lower $q_L$ is compared $q_H$, the more pronounced the reference-shift-induced *premium effect*.

### 4.3 Making Ad Content Endogenous

In this section, we present the advertising strategies that emerge in equilibrium. We observe a wide range of equilibria depending on the parameter space. Under certain conditions, there exists an equilibrium where the two competing firms adopt the same ad strategies, an equilibrium where the firms diverge in their ad strategies, and some where one of the firms foregoes advertising altogether. We first state the equilibrium outcome as a proposition.

**Proposition 3** (Advertising Equilibrium). Let the thresholds $\bar{\alpha}, \alpha, \bar{\alpha}, \bar{q}_L, \hat{q}_L$, and $\tilde{\gamma}$ be as defined in the proof (in Section A3.9 in the appendix).
I. If $\alpha \leq \tilde{\alpha}$ and $\gamma > \max \left[ \tilde{\gamma}, \frac{k}{v(q_L)} \right]$, then both firms choose valuation-shifting advertising;

II. if $\alpha > \tilde{\alpha}, q_L \leq \tilde{q}_L$, and $\gamma > \frac{k}{v(q_L - \delta)}$, then Firm $H$ chooses reference-shifting advertising and Firm $L$ chooses valuation-shifting advertising;

III. if $q_L > \tilde{q}_L$ and $\frac{k}{v(q_L)} \leq \gamma \leq \tilde{\gamma}$, then Firm $H$ does not advertise and Firm $L$ chooses valuation-shifting advertising;

IV. if $\alpha > \tilde{\alpha}, q_L \leq \tilde{q}_L$, and $\gamma \leq \frac{k}{v(q_L - \delta)}$, then Firm $H$ chooses reference-shifting advertising and Firm $L$ does not advertise;

V. if $\alpha \leq \min[\tilde{\alpha}, \alpha]$, and $\gamma \leq \frac{k}{v(q_L)}$, then Firm $H$ chooses valuation-shifting advertising and Firm $L$ does not advertise;

VI. otherwise, neither firm advertises.

The demarcations of the different equilibrium regions are illustrated in Figure 8.

Recall that in the monopoly case, reference-shifting advertising never arose in equilibrium. In contrast, Proposition 3 shows that, under certain conditions, reference-shifting advertising may be adopted in equilibrium. In particular, Firm $H$ chooses reference-shifting advertising when its rival’s quality level $q_L$ is sufficiently low (cases II and IV). The intuition is that reference-shifting advertising generates the premium effect, which allows Firm $H$ to capitalize on the enlarged
perceived quality differential (even though the absolute valuation of its product reduces). Reference-shifting advertising is especially profitable when $q_L$ is low because then the property of diminishing marginal utility of quality induces a disproportionately steeper devaluation for lower quality as the reference quality increases. Therefore, when $q_L$ is low, Firm $H$ chooses reference-shifting advertising in order to widen the perceived quality differential.

For larger $\gamma$, Firm $H$ may switch to valuation-shifting advertising. The reason is that when $\gamma$ is large, Firm $L$ chooses valuation-shifting advertising in order to lure Firm $H$’s consumers towards the low-quality product. And in fear of losing a large proportion of its high-valuation customers to Firm $L$, Firm $H$ adopts a defensive strategy by choosing valuation-shifting advertising to offset Firm $L$’s ad effect.

Another interesting implication of Proposition 3 is that Firm $H$ may forego such defensive advertising even though Firm $L$ poaches a fraction of Firm $H$’s consumers by means of valuation-shifting advertising. This occurs when $q_L$ is high and $\gamma$ is intermediate (case III). High $q_L$ implies not only that the *premium effect* generated by Firm $H$’s reference-shifting advertising is weak (see Proposition 2), but also that the *competition effect* associated with biasing the concentration of consumer types becomes more pronounced. That is, if $q_L$ is close to $q_H$ and consumer preferences become increasingly homogeneous, then the competition structure approaches Bertrand competition. In this case, even though $\gamma$ is high enough to incentivize Firm $L$ to choose valuation-shifting advertising, it is low enough such that Firm $H$ is better off relinquishing $\gamma$ fraction of its high-valuation consumers and maintaining high prices than intensifying competition by increasing the concentration of $h$-type consumers.

Finally, comparing Proposition 3 and Proposition 1 (which specifies the monopolist’s advertising strategy) reveals the intriguing result that there exist parametric regions for which either firm does not advertise in duopoly while it would have if it were a monopolist. For example, under the $(R, \emptyset)$ duopoly equilibrium (case IV) where Firm $L$ does not advertise, there exists a range of advertising costs $k$ for which Firm $L$ would advertise if it were a monopolist. The intuition lies in the *devaluation effect*; specifically, in the duopoly case, Firm $H$’s reference-shifting advertising reduces consumers’ valuation of $q_L$, which reduces Firm $L$’s return on valuation-shifting advertising,
and therefore it does not want to incur the advertising cost.

**Consumer Surplus**

In our framework, firms’ advertisements have a direct influence on consumers’ utility for quality; namely, advertisements shift the consumers’ taste and reference point for quality. Thus, we expect quality advertisements to have important implications for consumer surplus. As discussed in Dixit and Norman (1978), in persuasive advertising the utility function of the consumer changes because of the ads, and therefore deriving implications for consumer surplus involves making a choice between using pre-ad and post-ad utility functions for calculating post-ad consumer surplus. While there is no clear consensus on this point, we follow Kotowitz and Mathewson (1979) and the arguments in Bagwell (2005) and use post-ad utility functions for calculating post-ad consumer surplus.

Following this approach, the expected consumer surplus in equilibrium is

\[
\mathbb{E}[CS(a_H^*, a_L^*)] = \int \int_{(h \rightarrow q_H, l \rightarrow q_L)} \alpha(\theta_h v_H - p_H) + (1 - \alpha)(v_L - p_L) dF_H^*(p_H) dF_L^*(p_L) \\
+ \int \int_{(h \rightarrow q_L, l \rightarrow q_H)} \alpha(\theta_h v_L - p_L) + (1 - \alpha)(v_L - p_L) dF_H^*(p_H) dF_L^*(p_L),
\]

where \((h \rightarrow q_T, l \rightarrow q_T') \triangleq \{(p_H, p_L) : h\text{-type consumers buy } q_T, l\text{-type consumers buy } q_T'\}, v_T \triangleq v(q_T - \delta I_{a_H=R}), \alpha \triangleq \alpha^{a_H a_L}, \text{ and } (a_H^*, a_L^*) \text{ are the equilibrium advertising strategies characterized in the previous section.}

Is the expected consumer surplus greater when the two firms are more differentiated or less differentiated along the quality dimension? Standard economic theory suggests that less differentiation implies more intense price competition and, therefore, lower prices and higher consumer surplus. In the context of competitive advertising that we describe, however, we find that this need not be the case. While the standard reasoning rests on the premise that consumer valuation of quality remains constant, in our context, firms alter consumer valuation of quality through ads, which may affect consumer surplus. To illustrate, suppose the two firms are widely differentiated (e.g., \(q_L\) is low) such that competitive pressure is mild. In this case, Firm \(H\) chooses valuation-shifting advertising...
because the benefit of increasing its demand outweighs the cost of intensifying competition. And as a result of this ad campaign, a larger proportion of consumers derive higher utility from consuming a unit of quality than before observing the ad. This results in high consumer surplus.

On the other hand, consider the case where firms are mildly differentiated; i.e., competition is intense. Firm $H$ will not choose valuation-shifting advertising because increasing the concentration of high-valuation consumers would exacerbate competition even further. Thus, Firm $H$ foregoes valuation-shifting advertising, thereby mitigating competitive pressure at the expense of relinquishing a fraction of its consumers to Firm $L$. Compared to the previous case where Firm $H$’s advertising raised consumer valuation of quality, the absence of Firm $H$’s valuation-shifting advertising results in the average consumer deriving less utility from a unit of quality. In sum, even though competition is more intense for high $q_L$, the reduction in consumer valuation of quality brought about by Firm $H$’s withdrawal from valuation-shifting advertising may result in lower consumer surplus (see Figure 9). We summarize this result in the proposition below.

**Proposition 4.** The expected consumer surplus does not monotonically increase as the quality differentiation decreases. Specifically, under the conditions for which the equilibria $(V^h, V^l)$ and $(\emptyset, V^l)$ arise, there exists, for some fixed $q_H$, a pair $q^-_L < q^+_L$ such that the expected consumer surplus is smaller for $q_L = q^+_L$ than for $q_L = q^-_L$.

To conclude the discussion of the main model, we summarize our results on firms’ choices of valuation-shifting and reference-shifting advertising strategies obtained. We find that a monopolist, if it advertises, always chooses valuation-shifting advertising. In a duopoly scenario, the low-quality firm, if it advertises, also only chooses valuation-shifting advertising. In contrast, the high-quality
firm may choose valuation-shifting advertising or reference-shifting advertising, where the latter reduces the perceived absolute valuation of its product but increases its valuation relative to the low-quality product. Furthermore, either the low-quality or the high-quality firm may choose not to advertise at all (under conditions that this firm would if it were a monopolist). We now proceed to study extensions of the main model that show the robustness of these results.

5 Extensions

An assumption underlying the main model is that a firm can only choose one of reference-shifting and valuation-shifting advertising. In reality, however, both effects may be present in a single ad. For example, in Figure 1c, Verizon’s ad, which highlights its expansive network coverage, may not only shift the consumers’ reference quality for network coverage, but also enhance the consumer valuation for network coverage. Therefore, it is important to test the robustness of our main insights when advertisement contents are not as clear-cut as posited in the main model. We study this scenario in the first extension.

In the second extension, we consider the possibility of firms running multiple advertising campaigns with different contents. By allowing a firm to allocate a fixed advertising budget across the two types of content, we examine how the results of our main model change when firms can endogenously choose the mixture of advertisement content for a given advertising budget.

5.1 Multiple and Exogenous Ad Effects

In this section, we assume that a single ad has both reference-shifting and valuation-shifting effects. The relative strengths of each effect in the ad are exogenously determined by the parameter $\mu \in (0,1)$. Specifically, if Firm $H$ advertises, the proportion of high-valuation consumers increases by $(1 - \mu)\gamma$ and consumers’ reference quality increases by magnitude $\mu \delta$, while if Firm $L$ advertises the proportion of high-valuation consumers decreases by $(1 - \mu)\gamma$ and consumers’ reference quality increases by magnitude $\mu \delta$. Therefore, a small $\mu$ corresponds to ads that predominantly shift consumer valuations in the population, whereas a large $\mu$ corresponds to ads that predom-
inantly influence the consumers’ reference quality. Given these effects, the firms make a binary decision whether to advertise or not; i.e., \( a_T \in \{ A, \emptyset \} \). The rest of the model specifications remain unchanged.

Let \( \alpha^{a_H,a_L} \triangleq \alpha + (1 - \mu)\gamma \left( I_{\{a_H=A\}} - I_{\{a_L=A\}} \right) \), and \( v_T^{a_H,a_L} \triangleq v \left( q_T - \mu \delta \left( I_{\{a_H=A\}} + I_{\{a_L=A\}} \right) \right) \).

Then Firm \( H \)'s problem given Firm \( L \)'s advertising decision \( a_L \) is

\[
\max_{a_H \in \{A, \emptyset\}} E[\pi_H(a_H|a_L)], \quad (10)
\]

where \( E[\pi_H(a_H|a_L)] = \alpha^{a_H,a_L} \left( \theta_h \left( v_H^{a_H,a_L} - v_L^{a_H,a_L} \right) + (1 - \alpha^{a_H,a_L}) v_L^{a_H,a_L} - c \right) - k I_{\{a_H=A\}} \).

Similarly, Firm \( L \)'s problem is

\[
\max_{a_L \in \{A, \emptyset\}} E[\pi_L(a_L|a_H)], \quad (11)
\]

where \( E[\pi_L(a_L|a_H)] = (1 - \alpha^{a_H,a_L}) v_L^{a_H,a_L} - k I_{\{a_L=A\}} \).

From the above, we can derive the firms’ best responses as:

\[
a_T(a_{-T}) = \begin{cases} 
A & \text{if } E[\pi_T(A|a_{-T})] > E[\pi_T(\emptyset|a_{-T})], \\
\emptyset & \text{otherwise}.
\end{cases}
\]

We obtain the following proposition.

**Proposition 5.** Let \( \tilde{q}_L^e \) and \( \tilde{q}_L^c \) be as defined in the proof (in Section A.3.11 in the appendix).

I. If \( q_L \leq \tilde{q}_L^e \) and \( \gamma > \frac{k + (1 - \alpha) v(q_L - \delta \mu) - (1 - \alpha) v(q_L - 2\delta \mu)}{(1 - \mu)v(q_L - \delta \mu)} \), then both firms advertise;

II. if \( q_L \leq \tilde{q}_L^c \) and \( \gamma \leq \frac{k + (1 - \alpha) v(q_L - \delta \mu) - (1 - \alpha) v(q_L - 2\delta \mu)}{(1 - \mu)v(q_L - \delta \mu)} \), then only Firm \( H \) advertises;

III. if \( q_L > \tilde{q}_L^c \) and \( \alpha > \frac{k + (\gamma(\mu - 1) - 1) v(q_L - \delta \mu) + v(q_L)}{v(q_L) - v(q_L - \delta \mu)} \), then only Firm \( L \) advertises; and

IV. otherwise, neither firm advertises.

Figure 10 illustrates two equilibrium outcomes for different levels of \( \mu \). If \( \mu \) is small such that the valuation-shifting effect is dominant (Figure 10a), then we recover similar equilibrium outcomes as the original model — if \( \gamma \) is large, then both firms advertise, and if \( \gamma \) is intermediate, then
Firm $L$ advertises whereas Firm $H$ does not (in order to soften competition for the high-valuation consumers). On the other hand, if $\mu$ is large such that the reference-shifting effect is dominant, then Firm $L$ does not advertise altogether, while Firm $H$ advertises when $\delta$ is sufficiently large (Figure 10b). Intuitively, increasing the reference quality has an adverse effect on Firm $L$’s profit as it reduces consumer valuation for its product. Firm $H$, however, benefits from the reference-shifting effect due to the *premium effect*, which allows Firm $H$ to charge higher premiums. Broadly speaking, when ads are assumed to exogenously have both valuation-shifting and reference-shifting effects, the main underlying forces from the original model are still operational, but we obtain coarser equilibrium outcomes relative to the original model (e.g., larger parameter space for which neither firms advertise).

### 5.2 Multiple and Endogenous Ad Effects

In this section, we allow firms to endogenously choose how much of each effect (valuation- and reference-shifting) it wants to induce through its ads. To operationalize this, we allow each firm to allocate a fixed ad budget among the two forms of ads. To that end, suppose firms can invest $\lambda^V \in [0,1]$ proportion of their budget in valuation-shifting ads and $\lambda^R \in [0,1]$ proportion in reference-shifting ads with the constraint $\lambda^V + \lambda^R \leq 1$. The effect of investing $\lambda$ proportion of the budget to a certain type of ad is linearly related to the ad effect discussed in the previous
model. Specifically, an investment of $\lambda V$ in valuation-shifting advertising changes the proportion of $h$-type consumers by a magnitude of $\lambda V \gamma$, and an investment of $\lambda R$ in reference-shifting advertising increases the reference quality by a factor of $\lambda R \delta$. We assume a constant marginal cost of ads, $c(\lambda) = k \lambda$. (This specification is consistent with the cost function of the main model with $\lambda$’s constrained to $\{0, 1\}$.)

We first state the firms’ problems. Let $\alpha_{\lambda V H, \lambda V L} \triangleq \alpha + \gamma (\lambda V H - \lambda V L)$ and $v_{T H, T L} \triangleq v (q_T - \delta (\lambda R H + \lambda R L))$. Then Firm $H$’s problem given Firm $L$’s allocations $(\lambda V L, \lambda R L)$ is

$$\max_{0 \leq \lambda V H, \lambda R H \leq 1} \mathbb{E} [\pi_H (\lambda V H, \lambda R H) | \lambda V L, \lambda R L],$$

where $\mathbb{E} [\pi_H (\lambda V H, \lambda R H) | \lambda V L, \lambda R L] = \alpha_{\lambda V H, \lambda V L} \left( \theta_h \left( v_{H H, H L} - v_{L H, L L} \right) + (1 - \alpha_{\lambda V H, \lambda V L}) v_{L H, L L} - c \right) - k (\lambda V H + \lambda R H)$ and $\lambda V H + \lambda R H \leq 1$. Firm $L$’s problem is

$$\max_{0 \leq \lambda V L, \lambda R L \leq 1} \mathbb{E} [\pi_L (\lambda V L, \lambda R L) | \lambda V H, \lambda R H],$$

where $\mathbb{E} [\pi_L (\lambda V L, \lambda R L) | \lambda V H, \lambda R H] = (1 - \alpha_{\lambda V H, \lambda V L}) v_{L H, L L} - k (\lambda V L + \lambda R L)$ and $\lambda V L + \lambda R L \leq 1$.

We solve for Firm $L$’s best response to Firm $H$’s budget allocations, $\lambda V H$ and $\lambda R H$. First, it can be shown that investing any positive amount in reference-shifting advertising is strictly dominated. Therefore, we can replace $\lambda R L$ with 0 and simplify the firms’ problems. We obtain that Firm $L$’s optimal allocation to valuation-shifting advertising given Firm $H$’s allocation $(\lambda V H, \lambda R H)$ is

$$\lambda V L (\lambda V H, \lambda R H) = \begin{cases} 1 & \text{if } k \leq \gamma v (q_L - \lambda R H \delta), \\ 0 & \text{if } k > \gamma v (q_L - \lambda R H \delta). \end{cases}$$

In equilibrium, Firm $H$ rationally anticipates Firm $L$’s best response (14) and allocates its ad budget to maximize expected profit. Given Firm $L$’s best response, we solve for the SPNE by numerically computing Firm $H$’s best response.

Since the equilibrium outcome of this extension model is associated with continuous (vs. discrete)
decision variables, we plot how the equilibrium allocation levels vary with respect to exogenous variables. Figure 11 illustrates the equilibrium ad allocation levels as a function of $\gamma$. We see that, qualitatively, the equilibrium outcomes at extreme levels of $\gamma$ coincide with that of the original model. For example, when $\gamma$ is small, Firm $H$ invests all of its budget in reference-shifting advertising and Firm $L$ does not invest in either of the advertising types. And when $\gamma$ is large, both firms invest heavily in valuation-shifting advertising, mirroring the outcome from the original model. Therefore, broadly speaking, the underlying intuitions from the original model carry over.

Nevertheless, a distinct feature of the equilibrium outcome of this model is the partial budget allocation of Firm $H$ across different ads. When $\gamma$ is intermediate (neighborhood of $\gamma \approx 0.1$ in Figure 11), Firm $H$ distributes its budget partially across both types of advertising, and thereby takes advantage of two qualitatively distinct opportunities for improving its profit: it capitalizes on the premium effect generated by reference-shifting advertising, and increases its $h$-type demand base by valuation-shifting advertising. While the flexibility afforded by the opportunity to allocate budgets across different ad types “smoothens” the discrete advertising outcome from the original model, the main underlying forces and hence qualitative insights remain the same.

6 Conclusions and Discussion

We study advertising strategies of vertically differentiated competing firms. We consider two qualitatively distinct forms of advertising effects: (i) valuation-shifting effect, that changes the quality valuations of consumers (by inducing a quality focus or a value focus in consumers’ minds), and (ii) reference-shifting effect, that changes the reference point with respect to which consumers evaluate
quality (by influencing the quality anchor against which consumers evaluate quality). We obtain a number of interesting insights regarding firms’ use of these two types of content in their ads.

We find that a monopolist, if it advertises, always chooses valuation-shifting advertising to increase the proportion of high-valuation consumers in the population. Importantly, the monopolist never chooses reference-shifting advertising, because increasing the reference quality only reduces the consumers’ absolute valuation of the product’s quality, which in turn reduces the monopolist’s profit.

Interestingly, this intuition does not carry over when competition is introduced. Our analysis of a duopoly situation reveals that a high-quality firm may choose reference-shifting advertising to increase the consumers’ reference quality — even though doing so lowers the perceived quality for the firm’s product, due to the property of diminishing marginal utility of quality, it enhances its valuation relative to the low-quality product. Our findings also show that the high-quality firm does not unequivocally favor increasing the proportion of high-valuation consumers by doing valuation-shifting advertising, despite the fact that it serves this segment in equilibrium, because this creates greater price competition with the low-quality firm. We find that the low-quality firm chooses valuation-shifting advertising and never reference-shifting advertising, as does a monopolist, but with two crucial differences — its valuation-shifting advertising increases the proportion of low-valuation consumers in the market, and it does not always advertise under parametric conditions where a monopolist would. Another counterintuitive insight that we find is that smaller differentiation among firms’ qualities, even though it enhances competition, may lead to lower consumer surplus.

Our work offers a number of testable hypotheses regarding firms’ choices of advertising content and strategy in quality differentiated markets. For instance, using the results in Proposition 3 (and the associated Figure 8), we can state the following hypotheses. First (using Figure 8a), as the quality reference point of consumers becomes easier to manipulate (in the model, \( \delta \) increases), a high-quality firm uses less valuation-shifting content and more reference-shifting content in its advertising, while a low-quality firm reduces its advertising. Second (using Figure 8b), if the quality reference point of consumers is relatively difficult to manipulate (small \( \delta \)), then as the effectiveness
of valuation-shifting ads increases ($\gamma$ increases) advertising strategies in the market shift from only the high-quality firm using valuation-shifting content to both firms using valuation-shifting content in their advertising. Third (comparing Figures 8a and 8b), as the firms become less differentiated in quality ($q_L$ increases while $q_H$ is fixed) firms prefer valuation-shifting content more as compared to reference-shifting content in their advertising. To test these hypotheses, one could compare advertising in markets corresponding to different values of the model parameters. For instance, to test the first hypothesis, one could compare advertising strategies in markets corresponding to small and large $\delta$, respectively.

Our work is a first step towards understanding persuasive advertising in vertically differentiated markets, and presents many opportunities for further research. For example, it would be interesting to investigate what happens when the reference-updating process is allowed to be dynamic. While our model assumes that reference quality is determined primarily by ad exposure, the literature on reference points suggests that prior consumption experiences may play an important role in reference point formation (Hardie et al. 1993; Kopalle and Winer 1996). Therefore, future extensions could explore how the interaction of ad content and consumption experience in the reference updating process affects firms’ advertising decisions. Another possibility is to analyze how quality decisions are affected by ad decisions that we study. While our model abstracts from firms’ quality choices by imposing them to be exogenously endowed, future work may consider a quality decision stage prior to the stage where firms choose their ad types.

References


Appendix

A1 Parameter Space for Duopoly Analysis

For ease of exposition, we suppress the superscripts of $q^{aH,aL}$ and $\alpha^{aH,aL}$ which index the first-stage ad decisions. To simplify our analysis, we restrict attention to the parameter space for which Firm $H$ is not always better off serving the whole market at a low price than serving $h$-type consumers at a high price. To derive a sufficient condition for this, we impose that for all non-dominated price range of Firm $L$ and for all first stage ad decisions, Firm $H$’s maximum attainable profit when it serves the $h$-type is greater than that when it serves the whole market.

Given Firm $L$’s price $p_L$, Firm $H$’s profit maximizing prices for each of these cases are, respectively, $p_{h-only} = \theta_h(v(q_H - q_r) - v(q_L - q_r)) + p_L$ and $p_{whole} = \min[v(q_H - q_r) - v(q_L - q_r) + p_L, v(q_H - q_r)]$. Thus, we want to impose that

$$\min_{p_L \in \mathcal{P}_L} \mathbb{E}[\pi_H(p_{h-only})] - \mathbb{E}[\pi_H(p_{whole})] \geq 0,$$

(A1)

where $\mathcal{P}_L$ denotes the set of Firm $L$’s non-dominated prices. Writing out the expression for $\mathbb{E}[\pi_H(p_{h-only})] - \mathbb{E}[\pi_H(p_{whole})]$, we obtain that the difference is equal to

$$\begin{cases} 
\alpha \left( \theta_h(v(q_H - q_r) - v(q_L - q_r)) + p_L - c \right) - (v(q_H - q_r) - v(q_L - q_r) + p_L) - c & \text{if } p_L \leq v(q_L - q_r), \\
\alpha \left( \theta_h(v(q_H - q_r) - v(q_L - q_r)) + p_L - c \right) - (v(q_H - q_r) - c) & \text{if } p_L > v(q_L - q_r).
\end{cases}$$

(A2)

And since this difference is (i) continuous in $p_L$, (ii) decreasing in $p_L$ for $p_L \leq v(q_L - q_r)$, and (iii) increasing in $p_L$ for $p_L > v(q_L - q_r)$, we obtain that the minimum is attained at $p_L = v(q_L - q_r)$, which upon substitution yields

$$\min_{p_L \in \mathcal{P}_L} \mathbb{E}[\pi_H(p_{h-only})] - \mathbb{E}[\pi_H(p_{whole})] = \alpha \left( \theta_h(v(q_H - q_r) - v(q_L - q_r)) + v(q_L - q_r) - c \right) - (v(q_H - q_r) - c).$$

(A3)
Finally, to ensure that \((A3) \geq 0\) for all \((a_H, a_L)\), we impose

\[
\min_{a_H, a_L} \alpha^{\alpha_H,\alpha_L} \left( \theta_h \left( v(q_H - q_r^{\alpha_H,\alpha_L}) - v(q_L - q_r^{\alpha_H,\alpha_L}) \right) + v(q_L - q_r^{\alpha_H,\alpha_L}) - c \right) - \left( v(q_H - q_r^{\alpha_H,\alpha_L}) - c \right) \geq 0,
\]

for which a sufficient condition is

\[
(\alpha - 2\gamma) (\theta_h(v(q_H) - v(q_L)) + v(q_L) - c) - v(q_H) + c \geq 0. \tag{A4}
\]

Next, we set an upper bound for \(c\), the marginal cost of producing high quality \(q_H\), such that Firm \(H\)’s margin is positive when Firm \(H\) chooses \(V^h\). This rules out the case where Firm \(H\) refrains from choosing \(V^h\) due to production cost constraints. To derive a sufficient condition, we first establish that given \((A4)\), Firm \(L\) never chooses \(R\). This follows immediately from the fact that Firm \(L\)’s subgame equilibrium profit is \(E[\pi_L(a_H, a_L) = (1 - \alpha)v(q_L - q_r) - kI\{a_L \neq \emptyset\}]\), which is decreasing in \(q_r\). Therefore, Firm \(L\) does not choose reference-shifting advertising as doing so would only increase \(q_r\) and consequently lower its profit.

Second, Firm \(H\)’s margin can be shown to be \(\theta_h(v(q_H - q_r) - v(q_L - q_r)) + (1 - \alpha)v(q_L - q_r) - c\). In particular, if \(a_H = V^h\), then Firm \(H\)’s margin is

\[
\theta_h(v(q_H) - v(q_L)) + \left( 1 - \left( \alpha + \gamma - \gamma I\{a_L = V^l\} \right) \right) v(q_L) - c,
\]

which attains its minimum when \(a_L \neq V^l\) and \(\gamma = \alpha\). Thus, Firm \(H\)’s smallest attainable margin when it chooses \(V^h\) is \(\theta_h(v(q_H) - v(q_L)) + (1 - 2\alpha)v(q_L) - c\). Finally, imposing positivity on this value yields the upper-bound for the marginal cost of producing high quality:

\[
c \leq \bar{c} \triangleq \theta_h(v(q_H) - v(q_L)) + (1 - 2\alpha)v(q_L). \tag{A5}
\]

In sum, we restrict our parameter space for duopoly analysis to \((\alpha - 2\gamma) (\theta_h(v(q_H) - v(q_L)) + v(q_L) - c) - v(q_H) + c \geq 0\) and \(c \leq \theta_h(v(q_H) - v(q_L)) + (1 - 2\alpha)v(q_L)\).
A2 Duopoly Demand Boundaries

The roman numerics below correspond to the region labels in Figure 4. For ease of exposition, we suppress the superscripts of $q_r^{aH, aL}$ and $\alpha^{aH, aL}$ which index the first-stage ad decisions.

I. $p_L \leq \theta_l v(q_L - q_r)$ implies $\theta_l v_L - p_L \geq 0$ and $\theta_h v(q_L - q_r) - p_L > \theta_l v(q_L - q_r) - p_L \geq 0$ so that all consumers prefer $q_L$ over the outside option. And since $p_H > p_L + \theta_h (v(q_H - q_r) - v(q_L - q_r)) > p_L + \theta_l (v(q_H - q_r) - v(q_L - q_r))$ implies $\theta_h v(q_L - q_r) - p_L > \theta_h v(q_H - q_r) - p_L$ and $\theta_l v(q_L - q_r) - p_L > \theta_l v(q_H - q_r) - p_H$, we obtain that all consumers prefer $q_L$ over $q_H$.

II. $p_L \leq \theta_l v(q_L - q_r)$ implies that all consumers prefer $q_L$ over the outside option, and $p_L + \theta_l (v(q_H - q_r) - v(q_L - q_r)) < p_H \leq p_L + \theta_h (v(q_H - q_r) - v(q_L - q_r))$ implies that $l$-type consumers prefer $q_L$ over $q_H$, whereas $h$-type consumers prefer $q_H$ over $q_L$.

III. $p_L > \theta_l v(q_L - q_r)$ implies the $l$-type consumers prefer the outside option over $q_L$, and $p_H > p_L + \theta_h (v(q_H - q_r) - v(q_L - q_r))$ implies the $h$-type prefers $q_L$ over $q_H$.

IV. $p_L > \theta_l v(q_L - q_r)$ and $\theta_l v(q_H - q_r) < p_H \leq p_L + \theta_h (v(q_H - q_r) - v(q_L - q_r))$ imply that $l$-type consumers prefer the outside option over $q_L$ and $q_H$, whereas the $h$-type consumers prefer $q_H$ over $q_L$.

V. $p_H \leq \min [p_L + \theta_l (v(q_H - q_r) - v(q_L - q_r)), \theta_l v(q_H - q_r)]$ implies that $l$-type consumers prefer $q_H$ over $q_L$ and the outside option. The inequality also implies $p_H \leq \theta_h (v(q_H - q_r) - v(q_L - q_r))$ (because $\theta_h > \theta_l$) which in turn implies that the $h$-type consumers prefer $q_H$ over $q_L$.

A3 Proofs

A3.1 Proof of Lemma 1

For ease of exposition, we suppress the superscripts of $q_r^{aH, aL}$ and $\alpha^{aH, aL}$ which index the first-stage ad decisions. We first show that there does not exist a pure strategy equilibrium. Suppose, towards a contradiction, there exists one, and denote it by $(p_H^*, p_L^*)$. Since all $p_L$ beyond $\theta_h v(q_L - q_r)$ lead to zero demand, they must be dominated. Thus, it must be that $p_L^* \in [0, \theta_h v(q_L - q_r))$. 

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Secondly, by the assumption (A4) in Section A1 in the appendix, Firm H will not lower its price from \( \theta_h(v(q_H - q_r) - v(q_L - q_r)) + p_L \) to \( \min[v(q_H - q_r) - v(q_L - q_r) + p_L, v(q_H - q_r)] \) to serve the whole market. Therefore, Firm H’s best response to \( p_L \) is

\[
p_H(p_L) = \theta_h(v(q_H - q_r) - v(q_L - q_r)) + p_L, \tag{A6}
\]

which is the highest price Firm H can charge the \( h \)-segment without inducing them to switch to Firm L. Given Firm H’s best response (A6), however, Firm L has an incentive to undercut its price and attract the \( \alpha \)-size \( h \)-segment. More generally, given (A4), Firm L has incentive to undercut Firm H so long as \( p_L > 0 \). Since this implies that the only equilibrium candidate is \( p_L = 0 \) and \( p_H = p_H(0) = \theta_h(v(q_H - q_r) - v(q_L - q_r)) \), this must constitute the hypothesized pure strategy equilibrium. In this case, however, Firm L can again increase its profit by unilaterally deviating to \( p_L = v(q_L - q_r) \). This positive deviation contradicts that \( (p_H^*, p_L^*) \) constitutes an equilibrium.

Next, we construct the unique mixed strategy equilibrium. To that end, we first establish the equilibrium support by eliminating dominated strategies. As explained above, Firm L’s price in the interval \( (\theta_h v(q_L - q_r), \infty) \) yields zero demand and is thus dominated. Now suppose there exists a non-empty set \( P^0_L \) of non-dominated prices on the interval \( (v(q_L - q_r), \theta_h v(q_L - q_r)) \). By non-emptiness, we can pick the largest non-dominated price for Firm L, which we denote by \( \bar{p}_L = \max \{ p_L : p_L \in P^0_L \} \). This implies the largest non-dominated price for Firm H can be at most \( p_H = \theta_h(v(q_H - q_r) - v(q_L - q_r)) + \bar{p}_L \): this is the price at which the \( h \)-type consumer is indifferent between buying from Firm L and Firm H; any higher price will yield zero demand. However, if \( p_H \leq \bar{p}_H \), then \( p_L = \bar{p}_L \) yields zero profit for Firm L, while \( p_L = v(q_L - q_r) \) yields a positive payoff of either \( v(q_L - q_r) \) (if \( \theta_h(v(q_H - q_r) - v(q_L - q_r)) + v(q_L - q_r) < p_h \leq \theta_h(v(q_H - q_r) - v(q_L - q_r)) + \bar{p}_L \) such that all consumers buy from Firm L) or \( (1 - \alpha)v(q_L - q_r) \) (if \( p_H \leq \theta_h(v(q_H - q_r) - v(q_L - q_r)) + v(q_L - q_r) \) such that \( l \)-type consumers buy from Firm L). In other words, \( p_L = v(q_L - q_r) \) dominates \( p_L = \bar{p}_L \).

This is a contradiction because \( \bar{p}_L \) was a non-dominated price. Therefore, it must be that the non-emptiness assumption of \( P^0_L \) is false; i.e., there does not exist non-dominated prices on the interval \( (v(q_L - q_r), \infty) \).

The remaining set of non-dominated prices is \([0, v(q_L - q_r)]\). But we can reduce Firm L’s support
further by arguing as follows: if \( p_L \in [0, v(q_L - q_r)] \), then Firm \( H \) sets price on the interval 
\([\max[c, \theta_h(v(q_H - q_r) - v(q_L - q_r))), \theta_h(v(q_H - q_r) - v(q_L - q_r)) + v(q_L - q_r)]\). This, in turn, implies 
that any price \( p_L \) below \((1 - \alpha)v(q_L - q_r)\) is dominated by \( p_L = v(q_L - q_r) \). To see this, note that 
when Firm \( L \) sets price \( p_L = v(q_L - q_r) \), it exclusively attracts the \( l \)-type consumers for a profit 
\((1 - \alpha)v(q_L - q_r)\), regardless of Firm \( H \)’s price. Thus, any prices that yield payoff less than 
\((1 - \alpha)v(q_L - q_r)\) will be dominated. On the other hand, the maximum profit that Firm \( L \) 
can receive for \( p_L \in [0, (1 - \alpha)v(q_L - q_r)] \), even if Firm \( L \) manages to attract the whole market, is 
\((1 - \alpha)v(q_L - q_r)\); hence the result.

Finally, combined with the assumption (A5) on Firm \( H \)’s marginal cost \( c \) in Section A1 in the 
appendix, the equilibrium supports of Firms \( H \) and \( L \) simplify, respectively, to 
\[ \mathcal{P}_H = [(1 - \alpha)v(q_L - q_r) + \theta_h(v(q_H - q_r) - v(q_L - q_r)), \theta_h(v(q_H - q_r) - v(q_L - q_r)) + v(q_L - q_r)], \]
\[ \mathcal{P}_L = [(1 - \alpha)v(q_L - q_r), v(q_L - q_r)]. \]

The equilibrium distribution functions and corresponding profits are derived from the standard 
mixed equilibrium indifference conditions over these equilibrium supports.

### A3.2 Proof of Proposition 1

We first derive the subgame equilibrium price. Suppose the monopolist decides to target the whole 
market with a low price. The participation conditions inducing purchase from both segments are 
\( \theta_h v(q - q_r^h) - p \geq 0 \) and \( v(q - q_r^h) - p \geq 0 \). Due to \( \theta_h > 1 \), however, the second inequality implies 
the first, rendering the first redundant. The monopolist’s problem is 
\[
\max 1 \cdot p \\
\text{subject to } v(q - q_r^h) - p \geq 0.
\]

This yields \( p_{\text{whole}}^* = v(q - q_r^h) \) and corresponding profit \( \pi_{\text{whole}}^* = v(q - q_r^h) \).

Alternatively, if the monopolist aims to serve only the \( h \)-segment with a high price, then the 
conditions for inducing only the \( h \)-segment to purchase are \( \theta_h v(q - q_r^h) - p \geq 0 \) and \( v(q - q_r^h) - p < 0 \).
Thus, the monopolist solves

\[
\max \alpha^a p \\
\text{subject to } \theta_h v(q - q_r^a) \geq p \geq v(q - q_r^a),
\]

which yields \(p_{h-\text{only}}^* = \theta_h v(q - q_r^a)\) and corresponding profit \(\pi_{h-\text{only}}^* = \alpha^a \theta_h v(q - q_r^a)\).

Finally, the monopolist compares \(\pi_{\text{whole}}^*\) and \(\pi_{h-\text{only}}^*\) and chooses whichever strategy yields the higher payoff. Thus, the monopolist’s profit can be summarized as

\[\pi^*(a) = \max [v(q - q_r^a), \alpha^a \theta_h v(q - q_r^a)] - k\mathbb{I}_{\{a \neq \emptyset\}}.\]  

(A7)

But since \(\frac{\partial \pi^*(a)}{\partial \alpha^a} \geq 0\) and \(\frac{\partial \pi^*(a)}{\partial q \theta_h} < 0\), it follows that so long as the cost of advertising \(k\) is not too large \((k \leq ((\alpha + \gamma)\theta_h - \max[1, \alpha\theta_h])v(q))\), the monopolist will choose advertising that increases \(\alpha\), i.e., \(V^h\).

### A3.3 Proof of Lemma 2

For ease of exposition, we suppress the superscripts of \(q_{r}^{a_H,a_L}\) and \(\alpha_{a_H,a_L}\) which index the first-stage ad decisions. Suppose \(\alpha\) increases to \(\alpha' = \alpha + \epsilon\). We want to show that, for all \(\epsilon > 0\),

\[F_H^*(p_H; \alpha) \leq F_H^*(p_H; \alpha + \epsilon).\]  

(A8)

To that end, consider the price intervals of

\[(-\infty, \theta_h(v(q_H - q_r) - v(q_L - q_r)) + (1 - \alpha)v(q_L - q_r)]\]  

(A9)

and

\[[\theta_h(v(q_H - q_r) - v(q_L - q_r)) + v(q_L - q_r), \infty).\]  

(A10)

Inequality (A8) holds trivially in these intervals because in the first interval, \(F_H^*(p_H; \alpha) = 0\) and \(F_H^*(p_H; \alpha + \epsilon) \geq 0\). If \(p_H\) is in the second interval, both function values are equal to 1. Now,
consider the interval

\[
(\theta_h(v(q_H - q_r) - v(q_L - q_r)) + (1 - \alpha)v(q_L - q_r), \theta_h(v(q_H - q_r) - v(q_L - q_r)) + v(q_L - q_r)). \quad (A11)
\]

For all \( p_H \) in this interval, we have

\[
F^*_H(p_H; \alpha + \epsilon) - F^*_H(p_H; \alpha) = \int^{\alpha + \epsilon} \frac{\partial F^*_H(p_H; \alpha)}{\partial \alpha} d\alpha.
\]

But since \( p_H < \theta_h(v(q_H - q_r) - v(q_L - q_r)) + v(q_L - q_r) \) and \( p_H > \theta_h(v(q_H - q_r) - v(q_L - q_r)) + (1 - \alpha)v(q_L - q_r) \geq \theta_h(v(q_H - q_r) - v(q_L - q_r)), \) the integrand is strictly positive. Therefore, the integral is also strictly positive. The proof for the stochastic dominance of Firm L’s price distribution is similar and is omitted.

A3.4 Proof of Lemma 3

It suffices to show that

\[
\min_c \mathbb{E}[\pi_H(V^h, V^I)] - \mathbb{E}[\pi_H(V^I, V^I)] \geq 0
\]

and

\[
\min_c \mathbb{E}[\pi_H(V^h, \emptyset)] - \mathbb{E}[\pi_H(V^I, \emptyset)] \geq 0.
\]

The first inequality would imply that for Firm H, \( V^h \) dominates \( V^I \) when Firm L chooses \( V^I \), and the second inequality would imply the same dominance when Firm L does not advertise. But both
of the inequalities hold because

\[
\mathbb{E}[\pi_H(V^h, V^l)] - \mathbb{E}[\pi_H(V^l, V^l)] = 2\gamma(-c + \theta_h v(q_H) - v(q_L)(2\alpha - 2\gamma + \theta_h - 1)) \\
\geq 2\gamma(-\bar{c} + \theta_h v(q_H) - v(q_L)(2\alpha - 2\gamma + \theta_h - 1)) \\
= 4\gamma^2 v(q_L) \\
\geq 0,
\]

and

\[
\mathbb{E}[\pi_H(V^h, \emptyset)] - \mathbb{E}[\pi_H(V^l, \emptyset)] = -2\gamma(c - \theta_h v(q_H) + v(q_L)(2\alpha + \theta_h - 1)) \\
\geq -2\gamma(\bar{c} - \theta_h v(q_H) + v(q_L)(2\alpha + \theta_h - 1)) \\
= 0.
\]

### A3.5 Proof of Lemma 4

\[
\frac{\partial \mathbb{E}[\pi_L]}{\partial \alpha} = -v(q_L - q_r) < 0 \quad \text{and} \quad \frac{\partial \mathbb{E}[\pi_L]}{\partial q_r} = -(1 - \alpha)v'(q_L - q_r) < 0
\]

respectively imply that for Firm \(L\), not advertising is preferred over \(V^h\), and not advertising is preferred over \(R\). Therefore, if \(\mathbb{E}[\pi_L(a_H, V^l)] - \mathbb{E}[\pi_L(a_H, \emptyset)] \geq 0\), which is equivalent to \(k < \gamma v(q_L - \delta 1_{a_H=R})\), then Firm \(L\) advertises and chooses \(V^l\).

### A3.6 Proof of Lemma 5

\[
\frac{\partial \mathbb{E}[\pi_L]}{\partial \alpha} = -v(q_L - q_r^{a_H,a_L}) < 0,
\]

which implies that as \(\alpha\) decreases, Firm \(L\)'s profit increases. For the second part, simplify the inequality \(\frac{\partial \mathbb{E}[\pi_H]}{\partial \alpha} > 0\).

### A3.7 Proof of Lemma 6

\[
\frac{\partial \mathbb{E}[\pi_L]}{\partial q_r} = -(1 - \alpha)v'(q_L - q_r) < 0.
\]

For the second part, simplify the inequality \(\frac{\partial \mathbb{E}[\pi_H]}{\partial q_r} > 0\).
A3.8 Proof of Proposition 2

The profitability of Firm $H$’s valuation-shifting advertising:

$$\frac{\partial^2 E[\pi_H]}{\partial q_L \partial \alpha} = \frac{\partial v(q_L)}{\partial q_L} \frac{\partial E[\pi_H]}{\partial \alpha} = -v'(q_L)(\theta_h - 1 + 2\alpha) < 0$$

$$\frac{\partial^2 E[\pi_H]}{\partial q_H \partial \alpha} = \frac{\partial v(q_H)}{\partial q_H} \frac{\partial E[\pi_H]}{\partial \alpha} = v'(q_H)\theta_h > 0$$

$$\frac{\partial^2 E[\pi_H]}{\partial q_L \partial q_H} = v(q_H) - v(q_L) > 0$$

$$\frac{\partial^2 E[\pi_H]}{\partial \alpha^2} = -2v(q_L) < 0$$

The profitability of Firm $H$’s reference-shifting advertising:

$$\frac{\partial^2 E[\pi_H]}{\partial q_L \partial q_r} = \alpha(\theta_h - 1 + \alpha)v''(q_L - q_r) < 0$$

$$\frac{\partial^2 E[\pi_H]}{\partial q_H \partial q_r} = -\alpha \theta_h v''(q_H - q_r) > 0$$

$$\frac{\partial^2 E[\pi_H]}{\partial \theta_h \partial q_r} = \alpha (v'(q_L - q_r) - v'(q_H - q_r)) > 0$$

$$\frac{\partial^2 E[\pi_H]}{\partial \alpha \partial q_r} = \theta_h (v'(q_L - q_r) - v'(q_H - q_r)) - (1 - 2\alpha)v'(q_L - q_r) > 0$$

$$\iff \theta_h (v'(q_L - q_r) - v'(q_H - q_r)) > (1 - 2\alpha)v'(q_L - q_r)$$

premium effect

The profitability of Firm $L$’s valuation-shifting advertising:

$$-\frac{\partial^2 E[\pi_L]}{\partial q_L \partial \alpha} = v'(q_L - \delta_{a_H=R}) > 0$$

A3.9 Proof of Proposition 3

For each of the equilibrium candidate tuples $(a_H, a_L)$ enumerated below, we derive non-deviation conditions.

1. $(V^h, V^l)$
\[\begin{align*}
\text{• } \mathbb{E}[\pi_H(V^h, V^l)] &> \mathbb{E}[\pi_H(R, V^l)]: \text{ The difference } \mathbb{E}[\pi_H(V^h, V^l)] - \mathbb{E}[\pi_H(R, V^l)] \text{ is strictly concave in } \alpha \text{ because its second derivative is } -2(v(q_L) - v(q_L - \delta)). \text{ And at the minimum value of } \alpha \text{ which is } \gamma, \mathbb{E}[\pi_H(R, V^l)] \text{ is zero because there is no demand. Combining the concavity of the difference with respect to } \alpha \text{ and the positivity of the difference at the minimum value of } \alpha, \text{ we obtain that there exists unique root } \tilde{\alpha} \in [\gamma, \infty] \text{ that solves } \\
\mathbb{E}[\pi_H(V^h, V^l)] = \mathbb{E}[\pi_H(R, V^l)]. \text{ Then, we have } \mathbb{E}[\pi_H(V^h, V^l)] > \mathbb{E}[\pi_H(R, V^l)] \text{ if and only if } \alpha < \tilde{\alpha}.\footnote{Note that we denote the case where a real-valued solution to } \\
\mathbb{E}[\pi_H(V^h, V^l)] = \mathbb{E}[\pi_H(R, V^l)] \text{ does not exist as } \tilde{\alpha} = \infty. \text{ Then, consistent with the stated result, this implies that if a solution does not exist, then } \mathbb{E}[\pi_H(V^h, V^l)] > \mathbb{E}[\pi_H(R, V^l)] \text{ if and only if } \alpha < \infty, \text{ which means the inequality holds for all } \alpha.\]
\end{align*}\]
\[ \mathbb{E}[\pi_H(\emptyset, V^I)] > \mathbb{E}[\pi_H(R, V^I)]; \quad q_L > \bar{q}_L \quad (\because \text{complement of previous case}) \]
\[ \mathbb{E}[\pi_H(\emptyset, V^I)] > \mathbb{E}[\pi_H(V^h, V^I)]; \quad \gamma < \bar{\gamma} \quad (\because \text{complement of previous case}) \]
\[ \mathbb{E}[\pi_L(\emptyset, V^I)] > \mathbb{E}[\pi_L(\emptyset, \emptyset)]; \quad \text{The difference } \mathbb{E}[\pi_L(\emptyset, V^I)] - \mathbb{E}[\pi_H(\emptyset, \emptyset)] \text{ is strictly increasing in } \gamma \text{ because its derivative is } v(q_L). \quad \text{Therefore, } \mathbb{E}[\pi_L(\emptyset, V^I)] > \mathbb{E}[\pi_H(\emptyset, \emptyset)] \text{ if and only if } \gamma > \frac{k}{v(q_L)}. \]

4. \((R, \emptyset)\)

\[ \mathbb{E}[\pi_H(R, \emptyset)] > \mathbb{E}[\pi_H(V^h, \emptyset)]; \quad \text{The difference } \mathbb{E}[\pi_H(R, \emptyset)] - \mathbb{E}[\pi_H(V^h, \emptyset)] \text{ is strictly convex in } \alpha \text{ because its second derivative is } 2(v(q_L) - v(q_L - \delta)) > 0. \quad \text{And at the minimum value of } \alpha, \text{ which is zero in this case, } \mathbb{E}[\pi_H(R, \emptyset)] = 0 < \mathbb{E}[\pi_H(V^h, \emptyset)]. \quad \text{Therefore, there exists a unique root } \hat{\alpha} > 0 \text{ such that } \mathbb{E}[\pi_H(R, \emptyset)] > \mathbb{E}[\pi_H(V^h, \emptyset)] \text{ if and only if } \alpha > \hat{\alpha}. \]
\[ \mathbb{E}[\pi_H(R, \emptyset)] > \mathbb{E}[\pi_H(\emptyset, \emptyset)]; \quad \text{The difference } \mathbb{E}[\pi_H(R, \emptyset)] - \mathbb{E}[\pi_H(\emptyset, \emptyset)] \text{ is strictly decreasing in } q_L \text{ because its derivative is } -\alpha(\theta_h - 1 + \alpha)(v'(q_L - \delta) - v'(q_L)) < 0. \quad \text{Therefore, } \mathbb{E}[\pi_H(R, \emptyset)] > \mathbb{E}[\pi_H(\emptyset, \emptyset)] \text{ if and only if } q_L < \bar{q}_L \text{ where } \bar{q}_L \in [0, \infty] \text{ is the unique root that solves } \mathbb{E}[\pi_H(R, \emptyset)] = \mathbb{E}[\pi_H(\emptyset, \emptyset)]. \]
\[ \mathbb{E}[\pi_L(R, \emptyset)] > \mathbb{E}[\pi_L(R, V^I)]; \quad \gamma < \frac{k}{v(q_L - \delta)} \quad (\because \text{complement of previous case}) \]

5. \((V^h, \emptyset)\)

\[ \mathbb{E}[\pi_H(V^h, \emptyset)] > \mathbb{E}[\pi_H(R, \emptyset)]; \quad \alpha < \hat{\alpha} \quad (\because \text{complement of previous case}) \]
\[ \mathbb{E}[\pi_H(V^h, \emptyset)] > \mathbb{E}[\pi_H(\emptyset, \emptyset)]; \quad \text{The difference } \mathbb{E}[\pi_H(V^h, \emptyset)] - \mathbb{E}[\pi_H(\emptyset, \emptyset)] \text{ is strictly decreasing in } \alpha \text{ because its derivative is } -2\gamma v(q_L). \quad \text{It follows that } \mathbb{E}[\pi_H(V^h, \emptyset)] > \mathbb{E}[\pi_H(\emptyset, \emptyset)] \text{ if and only if } \alpha < \bar{\pi} \text{ where } \bar{\pi} \in [0, \infty] \text{ is the unique root that solves } \mathbb{E}[\pi_H(V^h, \emptyset)] = \mathbb{E}[\pi_H(\emptyset, \emptyset)]. \]
\[ \mathbb{E}[\pi_L(V^h, \emptyset)] > \mathbb{E}[\pi_L(V^h, V^I)]; \quad \gamma < \frac{k}{v(q_L)} \quad (\because \text{complement of previous case}) \]

6. \((\emptyset, \emptyset)\)

\[ \mathbb{E}[\pi_H(\emptyset, \emptyset)] > \mathbb{E}[\pi_H(V^h, \emptyset)]; \quad \alpha > \bar{\alpha} \quad (\because \text{complement of previous case}) \]
\[ \mathbb{E}[\pi_H(\emptyset, \emptyset)] > \mathbb{E}[\pi_H(R, \emptyset)]; \quad \text{The difference } \mathbb{E}[\pi_H(\emptyset, \emptyset)] - \mathbb{E}[\pi_H(\emptyset, \emptyset)] \text{ is strictly decreasing in } q_L \text{ because its derivative with respect to } q_L \text{ is } -(\theta_h - 1 + \alpha)(v'(q_L - \delta) - v'(q_L)) < 0 \]
(note that we have used \( \theta_h > 1 \) and \( v''(\cdot) < 0 \) here). Therefore, \( \mathbb{E}[\pi_H(0, 0)] \geq \mathbb{E}[\pi_H(R, 0)] \) if and only if \( q_L > \hat{q}_L \) where \( \hat{q}_L \) is the unique root of \( \mathbb{E}[\pi_H(R, 0)] = \mathbb{E}[\pi_H(0, 0)] \).

- \( \mathbb{E}[\pi_L(0, 0)] > \mathbb{E}[\pi_L(0, V^l)] \): The difference \( \mathbb{E}[\pi_L(0, 0)] - \mathbb{E}[\pi_L(0, V^l)] \) is strictly decreasing in \( \gamma \) because its derivative is \( -v(q_L) \). Therefore, \( \mathbb{E}[\pi_L(0, 0)] > \mathbb{E}[\pi_L(0, V^l)] \) if and only if 
  \[ \gamma < \frac{k}{v(q_L)}. \]

\section*{A3.10 Proof of Proposition 4}

Consider the expected consumer surplus given in Equation (9). The integration bounds can be written more explicitly as follows:

\begin{equation}
(9) = \int_{1-\alpha}^{v_L} \int_{\theta_h(v_H - v_L) + p_L}^{\theta_h(v_H - v_L) + p_L} \alpha(\theta_h v_H - p_H) + (1 - \alpha)(v_L - p_L) dF_H^*(p_H) dF_L^*(p_L) \tag{A12}
\end{equation}

First, we have \( \frac{\partial^2 E[CS]}{\partial \alpha^2} = \frac{(c+\theta_h(v_L-v_H))^2}{(\alpha-1)(c-\theta_h v_H + v_L(\alpha+\theta_h-1))^2} \geq 0 \) which implies \( \frac{\partial^2 E[CS]}{\partial \alpha^2} \) is minimized at \( \alpha = 0 \). But at \( \alpha = 0 \), \( \frac{\partial^2 E[CS]}{\partial \alpha^2} = 1 + \frac{v_L}{\theta_h(v_H - v_L) + v_L - c} > 0 \). Therefore, \( \frac{\partial^2 E[CS]}{\partial \alpha^2} > 0 \). This in turn implies that \( \frac{\partial E[CS]}{\partial \alpha} \) is minimized at \( \alpha = 0 \). But at \( \alpha = 0 \), \( \frac{\partial E[CS]}{\partial \alpha} = \theta_h - 1 > 0 \). Therefore, \( \frac{\partial E[CS]}{\partial \alpha} > 0 \).

Secondly, let \( \bar{q}_L \) be the \( q_L \) that makes Firm \( H \) indifferent between choosing valuation-shifting advertising and not advertising, provided Firm \( L \) chooses valuation-shifting advertising. Note that such \( \bar{q}_L \) exists by our hypothesis that the strategy profiles \((V^h, V^l)\) and \((0, V^l)\) arise in the equilibrium region of interest. And it is unique because

\begin{align*}
\frac{\partial}{\partial q_L} (\mathbb{E}[\pi_H(V^h, V^l)] - \mathbb{E}[\pi_H(0, V^l)]) &= \frac{\partial v(q_L)}{\partial q_L} \frac{\partial}{\partial v(q_L)} (\mathbb{E}[\pi_H(V^h, V^l)] - \mathbb{E}[\pi_H(0, V^l)]) \\
&= -v'(q_L) \gamma (\theta_h - 1 + 2\alpha - \gamma) \\
&< 0,
\end{align*}

which implies that \( \mathbb{E}[\pi_H(V^h, V^l)] > \mathbb{E}[\pi_H(0, V^l)] \) if and only if \( q_L < \bar{q}_L \).

Therefore, at \( q_L \) arbitrarily close to but less than \( \bar{q}_L \), Firm \( H \) chooses \( V^h \) which increases consumers' valuation of quality, and at \( q_L \) arbitrarily close to but greater than \( \bar{q}_L \), Firm \( H \) withdraws from \( V^h \).
and the proportion of high-valuation consumers decreases by $\gamma$. And since, all else equal, $\mathbb{E}[CS]$ is continuous in $\alpha$ and $\frac{\partial \mathbb{E}[CS]}{\partial \alpha} > 0$, the discrete decline in $\alpha$ due to Firm $H$’s shift in advertising regime from $V^h$ to $\emptyset$, which is induced by higher $q_L$, implies a discrete drop in consumer surplus as well. Denote this magnitude of surplus decline by $\Delta$. Since $\mathbb{E}[CS]$ is continuous in the neighborhood of $q_L = \bar{q}_L$ (except at $\bar{q}_L$), we can find for any $\epsilon > 0$ (in particular, $\epsilon < \Delta$), a pair $(\bar{q}_L^-, \bar{q}_L^+)$ such that (i) $\bar{q}_L^- < \bar{q}_L < \bar{q}_L^+$, and (ii) $\mathbb{E}[CS(\bar{q}_L^-)] - \mathbb{E}[CS(\bar{q}_L^+)] > \Delta - \epsilon > 0$. This completes the proof.

A3.11 Proof of Proposition 5

As we did in the proof of Proposition 3, we write the non-deviation conditions for each equilibrium candidate enumerated below.

I. $(A, A)$:

- $\mathbb{E}[\pi_H(A, A)] > \mathbb{E}[\pi_H(\emptyset, A)]$: The difference $\mathbb{E}[\pi_H(A, A)] - \mathbb{E}[\pi_H(\emptyset, A)]$ is strictly decreasing in $q_L$ because

$$\frac{\partial}{\partial q_L} \mathbb{E}[\pi_H(A, A)] - \mathbb{E}[\pi_H(\emptyset, A)] = \alpha(\theta_h - 1 + \alpha) \left( v'(q_L - \mu\delta) - v'(q_L - 2\mu\delta) \right)$$

$$- \alpha \gamma (1 - \mu) v'(q_L - \mu\delta)$$

$$- \gamma (1 - \mu) (\theta_h - 1 + \alpha - \gamma (1 - \mu)) v'(q_L - \mu\delta)$$

$$< 0$$

Therefore, if $\bar{q}_L^c$ is the unique $q_L$ that solves $\mathbb{E}[\pi_H(A, A)] = \mathbb{E}[\pi_H(\emptyset, A)]$, then $\mathbb{E}[\pi_H(A, A)] > \mathbb{E}[\pi_H(\emptyset, A)]$ if and only if $q_L < \bar{q}_L^c$.

- $\mathbb{E}[\pi_L(A, A)] > \mathbb{E}[\pi_L(A, \emptyset)]$: The difference $\mathbb{E}[\pi_L(A, A)] - \mathbb{E}[\pi_L(\emptyset, A)]$ is strictly increasing in $\gamma$ because $\frac{\partial}{\partial \gamma} \mathbb{E}[\pi_L(A, A)] - \mathbb{E}[\pi_L(\emptyset, A)] = (1 - \mu) v(q_L - \mu\delta) > 0$. Therefore, $\mathbb{E}[\pi_L(A, A)] > \mathbb{E}[\pi_L(\emptyset, A)]$ if and only if $\gamma > \frac{k + (1 - \alpha) v(q_L - \mu\delta) - (1 - \alpha) v(q_L - 2\mu\delta)}{(1 - \mu) v(q_L - \mu\delta)}$.

II. $(A, \emptyset)$:

- $\mathbb{E}[\pi_H(A, \emptyset)] > \mathbb{E}[\pi_H(\emptyset, \emptyset)]$: The difference is strictly decreasing in $q_L$ because its derivative is $-\gamma (1 - \mu)(\alpha + \gamma (1 - \mu) + \theta_h - 1) v'(q_L - \mu\delta) - \alpha \gamma (1 - \mu) v'(q_L - \delta\mu) + \alpha (\alpha + \theta_h - \mu\delta) - \alpha \gamma (1 - \mu) v'(q_L - \delta\mu) + \alpha (\alpha + \theta_h - \mu\delta)$.
1) \((v'(q_L) - v'(q_L - \delta \mu)) < 0\). Therefore, if \(\hat{q}_L^e\) is the unique \(q_L\) that solves \(E[\pi_H(A, \emptyset)] = E[\pi_H(\emptyset, \emptyset)]\), then \(E[\pi_H(A, \emptyset)] > E[\pi_H(\emptyset, \emptyset)]\) if and only if \(q_L < \hat{q}_L^e\).

• \(E[\pi_L(A, \emptyset)] > E[\pi_L(A, A)]\): \(\gamma < \frac{k+1-v(q_L)-v(q_L-\mu \delta)}{v(q_L-\delta \mu)}\) (\(\vdash\) complement of previous case)

III. (\(\emptyset, A\)):

• \(E[\pi_H(\emptyset, A)] > E[\pi_H(A, A)]\): \(q_L > \hat{q}_L^e\) (\(\vdash\) complement of previous case)

• \(E[\pi_L(\emptyset, A)] > E[\pi_L(\emptyset, \emptyset)]\): The difference \(E[\pi_L(\emptyset, A)] - E[\pi_L(\emptyset, \emptyset)]\) is strictly increasing in \(\alpha\) because its derivative is \(v(q_L) - v(q_L - \mu \delta) > 0\). Therefore, \(E[\pi_L(\emptyset, A)] > E[\pi_L(\emptyset, \emptyset)]\) if and only if \(\alpha > \frac{k+\gamma k-1-v(q_L-\mu \delta)+v(q_L)}{v(q_L)-v(q_L-\delta \mu)}\).

IV. (\(\emptyset, \emptyset\)):

• \(E[\pi_H(\emptyset, \emptyset)] > E[\pi_H(A, \emptyset)]\): \(q_L > \hat{q}_L^e\) (\(\vdash\) complement of previous case)

• \(E[\pi_L(\emptyset, \emptyset)] > E[\pi_L(\emptyset, A)]\): \(\alpha < \frac{k+\gamma k-1-v(q_L-\mu \delta)+v(q_L)}{v(q_L)-v(q_L-\delta \mu)}\) (\(\vdash\) complement of previous case)

A4 Online Experiment

In the experiment, subjects were exposed to ads of robot vacuum cleaners of two different quality levels, after which they were asked to indicate their WTP for the low-quality robot vacuum cleaner. Subjects were randomly assigned to either of the two treatment conditions: exposure to the low-quality product ad (Condition 1) or exposure to the high-quality product ad (Condition 2). The ad treatments that the subjects received were presented alongside an unrelated article about animal conservation (see Figures A1 and A2). The evaluation task involved subjects indicating their WTP for the low-quality robot vacuum cleaner on a price range of $0 to $1,000. See Figures A1, A2 and A3 for screenshots. We removed 6 responses which were submitted in less than 20 seconds. After this, we had 31 subjects in Condition 1 and 37 subjects in Condition 2. In Condition 1, the average WTP for the low-quality product was $181.25 and in Condition 2, the average WTP.
for the low-quality product was $123.81; a standard test shows that the numbers are significantly different ($t = 2.27, p < 0.03$). The results are plotted in Figure A4.

**Figure A1: Condition 1: Low-Quality Product Ad Treatment**

**Declining Number of Tigers**

The population of tigers is believed to have declined by 95 percent in the last century. Tigers continue to face challenges imposed by poaching, retributive killings and habitat loss. Tiger bone is also in high demand for traditional medicines in China and some other parts of the world, often based on mistaken beliefs, or weak evidence for their effectiveness. IUCN is the world's oldest environmental organization, working around the world. Periodically, they produce the IUCN Red List of Threatened Species to highlight species that are extinct or extinct in the wild, critically endangered, endangered or vulnerable. Their data suggests that the global tiger population has declined to an estimated range from 3,402-5,140 tigers, revised down from estimates of 5,000 to 7,000 made a few years earlier.

Figure A2: Condition 2: High-Quality Product Ad Treatment

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8To address outliers in the sample, we winsorized the data using 5% quintiles on either end of the sample tails, a technique widely used in the finance and accounting literature (e.g., Mendenhall 1991, Hadlock and Pierce 2010).
Now, imagine you came across the robot vacuum below at your local retailer.

MARAX 35 Robot Vacuum

- Cleans floors
- 1 year warranty

Please indicate your maximum willingness-to-pay (in $US) for this robot vacuum using the following scale.

Figure A3: Product Evaluation Task

Figure A4: Experiment Results